

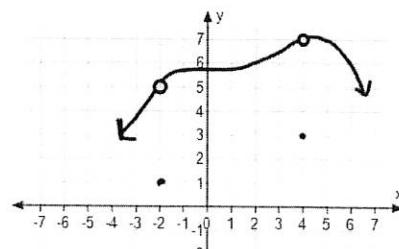
## Warm-Up

1) What is a limit? A limit is the y-value as you approach an x-value.

2) Use the graph to find each of the following.

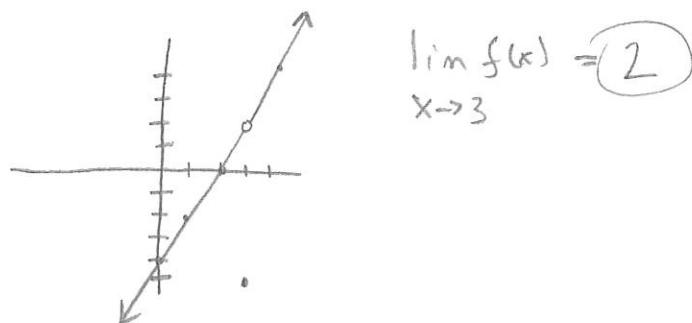
a)  $\lim_{x \rightarrow 4} f(x) = 7$

b)  $f(4) = 3$



3) Graph the function:  $\begin{cases} 2x - 4 & \text{if } x \neq 3 \\ -5 & \text{if } x = 3 \end{cases}$

then find  $\lim_{x \rightarrow 3} f(x)$



**EXAMPLE 1** Finding Limits of Constant Functions

Find the following limits:

a.  $\lim_{x \rightarrow 4} 7$

b.  $\lim_{x \rightarrow 0} (-5)$

(7)

(-5)

**EXAMPLE 2** Finding Limits of the Identity Function

Find the following limits:

a.  $\lim_{x \rightarrow 7} x$

b.  $\lim_{x \rightarrow -\pi} x$

$\lim_{x \rightarrow 7} x = 7$

$\lim_{x \rightarrow -\pi} x = -\pi$

**EXAMPLE 3** Finding the Limit of a Sum

Find:  $\lim_{x \rightarrow -4} (x + 9)$ .

$$\lim_{x \rightarrow -4} x + \lim_{x \rightarrow -4} 9 = -4 + 9 = 5$$

**EXAMPLE 4** Finding the Limit of a Difference

Find:  $\lim_{x \rightarrow 5} (12 - x)$ .

$$\lim_{x \rightarrow 5} 12 - \lim_{x \rightarrow 5} x = 12 - 5 = 7$$

**EXAMPLE 5** Finding the Limit of a Product

Find:  $\lim_{x \rightarrow 5} (-6x)$ .

$$\lim_{x \rightarrow 5} (-6) \cdot \lim_{x \rightarrow 5} x = -6(5) = -30$$

**EXAMPLE 6** Finding Limits Using Properties of Limits

Find the following limits:

a.  $\lim_{x \rightarrow -3} (7x - 4)$

$$\lim_{x \rightarrow -3} 7 \cdot \lim_{x \rightarrow -3} x - \lim_{x \rightarrow -3} 4$$

$$7(-3) - 4$$

$$-21 - 4$$

$$-25$$

b.  $\lim_{x \rightarrow 5} 6x^2$

$$\lim_{x \rightarrow 5} 6 \cdot \lim_{x \rightarrow 5} x \cdot \lim_{x \rightarrow 5} x$$

$$6 \cdot 5 \cdot 5$$

$$150$$

**EXAMPLE 7** Finding the Limit of a Monomial

Find:  $\lim_{x \rightarrow 2} (-6x^4)$ .

$$\lim_{x \rightarrow 2} -6 \cdot 2^4 = -6 \cdot 16 = \boxed{-96}$$

**EXAMPLE 8** Finding the Limit of a Polynomial

Find:  $\lim_{x \rightarrow 3} (4x^3 + 2x^2 - 6x + 5)$ .

$$4(3)^3 + 2(3)^2 - 6(3) + 5$$

$$4(27) + 2(9) - 18 + 5$$

$$108 + 18 - 18 + 5$$

113

**EXAMPLE 9** Finding the Limit of a Power

Find:  $\lim_{x \rightarrow 5} (2x - 7)^3$ .  $= \left[ \lim_{x \rightarrow 5} (2x - 7) \right]^3 = (2 \cdot 5 - 7)^3 = 3^3 = \boxed{27}$

**EXAMPLE 10** Finding the Limit of a Root

Find:  $\lim_{x \rightarrow -2} \sqrt{4x^2 + 5}$ .  $= \sqrt{\lim_{x \rightarrow -2} 4x^2 + 5} = \sqrt{4(-2)^2 + 5} = \sqrt{16 + 5} = \boxed{\sqrt{21}}$

**EXAMPLE 11** Finding the Limit of a Quotient

Find:  $\lim_{x \rightarrow 1} \frac{x^3 - 3x^2 + 7}{2x - 5}$ .

Check the denominator first...

$$\lim_{x \rightarrow 1} 2x - 5 = 2(1) - 5 = -3 \leftarrow \text{As long as we didn't get } \underline{\text{zero}} \text{ then we can use the quotient rule.}$$

$$\frac{\lim_{x \rightarrow 1} x^3 - 3x^2 + 7}{\lim_{x \rightarrow 1} 2x - 5} = \frac{1^3 - 3 \cdot 1^2 + 7}{-3} = \frac{5}{-3} = \boxed{-\frac{5}{3}}$$