

Semester 1 Final Exam Review

Constructed Response- Calculator Allowed

$$\frac{\text{Factors of 18}}{\text{Factors of 1}} = \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$$

1. Given $x^4 + x^3 + 7x^2 + 9x - 18$,

- List all possible rational zeros
- Using the graph, synthetic division, and factoring/quadratic formula, find all zeros of the function (you must show work for each zero).

- You can use the graph to identify the x-intercepts, then use those in synthetic division.

- The graph has two x-int: -2 & 1 but the equation is x^4 , so there must be two additional zeros that are imaginary.

$$\begin{array}{r|rrrrrr} 1 & 1 & 1 & 7 & 9 & -18 \\ + & 0 & 1 & 2 & 9 & 18 \\ \hline & & 2 & 9 & 18 & 0 \end{array}$$

$x=1$ is a zero

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 9 & 18 \\ + & & -2 & -5 & -9 \\ \hline & 1 & 0 & 4 & 9 \end{array}$$

$x=-2$ is a zero

$$\begin{aligned} x^2 + 9 &= 0 \\ -9 & \quad -9 \\ \hline \sqrt{x^2} &= \sqrt{-9} \\ x &= \pm 3i \end{aligned}$$

Zeros: $x = 1, -2, \pm 3i$

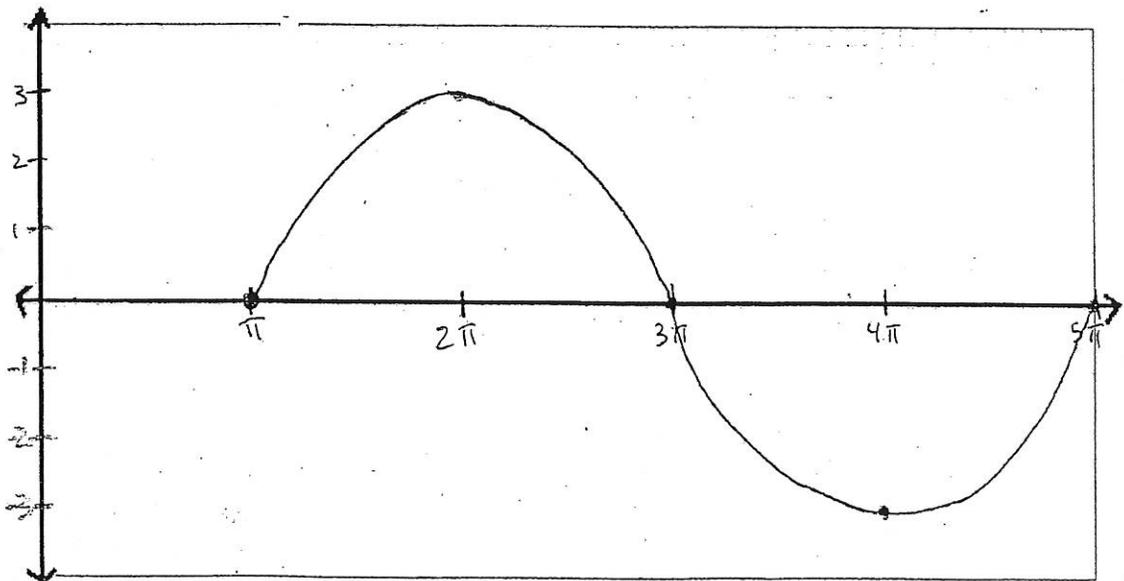
2. Graph the function $f(x) = 3\sin\left(\frac{1}{2}(x-\pi)\right) = 3\sin\left(\frac{1}{2}x - \frac{\pi}{2}\right)$

- Period 4π
- amp 3
- phase shift π
- vert shift None

$$\frac{P}{4} = \frac{4\pi}{4} = \pi$$

- domain \mathbb{R} or $[\pi, 5\pi]$
- range $[-3, 3]$

X	Y
π	0
2π	3
3π	0
4π	-3
5π	0



3. Prove the following identity $\frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \sin t + \cos t$

$$\frac{\sin t}{\sin t / \cos t} + \frac{\cos t}{\cos t / \sin t}$$

$$\cancel{\sin t} \cdot \frac{\cos t}{\cancel{\sin t}} + \cancel{\cos t} \cdot \frac{\sin t}{\cancel{\cos t}}$$

$$\cos t + \sin t$$

$$\sin t + \cos t = \sin t + \cos t$$

4. Solve the following equation for all real numbers $\cos \frac{2\alpha}{3} = -1$

$$\cos \theta = -1$$

$$\theta = \pi + 2n\pi$$

~~2~~

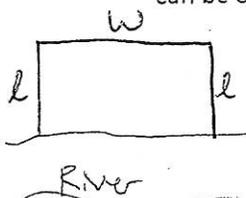
$$\frac{3 \cdot 2\alpha}{2 \cdot 3} = (\pi + 2n\pi) \cdot \frac{3}{2}$$

$$\alpha = \frac{3\pi}{2} + 3n\pi$$

Semester 1 Final Exam Review

Multiple Choice-Calculator Allowed

1. You have 600 feet of fencing to enclose a rectangular plot that borders a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?



$$P = 2l + w$$

$$A = l \cdot w$$

$$600 = 2l + w$$

$$(600 - 2l) = w$$

$$A = l(600 - 2l)$$

$$A = 600l - 2l^2$$

Need to maximize

2. Convert $\frac{5\pi}{12}$ to degrees.

$$\frac{5\pi}{12} \cdot \frac{180}{\pi} = \frac{5(180)}{12} = 75^\circ$$

$$x = \frac{-b}{2a} = \frac{-(600)}{2(-2)} = \frac{-600}{-4} = 150 \leftarrow \text{plug in to find } w$$

$$w = 600 - 2(150) = 300 \quad (150, 300) \text{ Max pt}$$

3. Use a calculator to evaluate $\csc \frac{\pi}{12} = \frac{1}{\sin \frac{\pi}{12}} = 3.8637$

$$\csc \theta = \frac{1}{\sin \theta}$$

* Calc in radians

$$A = (150)(300) = 45,000 \text{ ft}^2$$

4. Find the arc length of the intercepted arc in a circle of radius 13 in. and central angle of 110° .

$$S = r\theta \text{ in radians}$$

$$S = 13 \left(110 \cdot \frac{\pi}{180} \right) \approx 7.94 \text{ in}$$

5. A building that is 21 meters tall casts a shadow 25 meters long. Find the angle of elevation of the sun to the nearest degree. ** This is nearest degree. on the constructed response now!*



$$\tan \theta = \frac{21}{25} \quad \tan^{-1} \left(\frac{21}{25} \right) = 40^\circ$$

6. Solve the equation $\frac{2 \tan 2x}{2} = -6.2154$ for all real numbers.

$$\tan 2x = -3.1077$$

$$\tan \theta = -3.1077$$

$$\theta = -1.2595 + n\pi$$

$$\frac{2x}{2} = \frac{-1.2595 + n\pi}{2}$$

$$x = -0.6297 + \frac{n\pi}{2}$$

or

$$x = 5.6535 + \frac{n\pi}{2}$$

$$2\pi + (-0.6297)$$

MULTIPLE CHOICE SECTION

Non-Calculator:

What needs to be excluded?

$\frac{1}{\#}$ ← makes this 0 $\sqrt{x+7}$ ← makes this negative

7. Find the domain of the function $f(x) = \frac{1}{3\sqrt{x+7}}$

$x > -7$

Domain $(-7, \infty)$

8. Determine the domain of $f+g$, $f-g$, fg , and f/g of $f(x) = \sqrt{x+4}$ $g(x) = \sqrt{x-1}$

$f+g = \sqrt{x+4} + \sqrt{x-1}$ Domain: $[1, \infty)$

$f-g = \sqrt{x+4} - \sqrt{x-1}$ Domain: $[1, \infty)$

$f \cdot g = \sqrt{(x+4)(x-1)} = \sqrt{x^2+3x-4}$ Domain: $[1, \infty)$

$\frac{f}{g} = \frac{\sqrt{x+4}}{\sqrt{x-1}}$

Domain $(1, \infty)$

9. Find $(f \circ g)(x)$ when $f(x) = x^2 + 1$, $g(x) = \sqrt{2-x}$ $(f \circ g)(x) = (\sqrt{2-x})^2 + 1 = 2 - x + 1 = 3 - x$

10. Write an equation for the inverse function, $f^{-1}(x)$ when $f(x) = \frac{2x+1}{x-3}$
Switch x & y & solve

$x = \frac{2y+1}{y-3}$

$x(y-3) = 2y+1$

$xy - 3x = 2y + 1$
 $-2y + 3x - 2y + 3x$

$xy - 2y = 3x + 1$

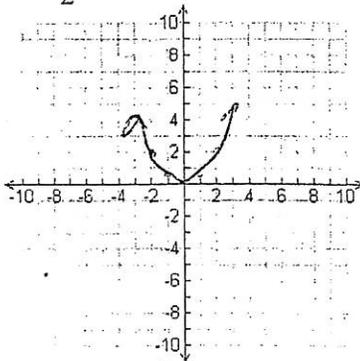
$y(x-2) = 3x+1$
 $\frac{y(x-2)}{x-2} = \frac{3x+1}{x-2}$

$y = \frac{3x+1}{x-2}$

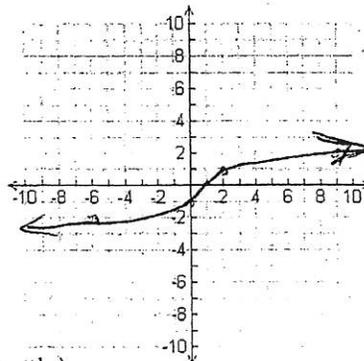
11. What transformation(s) of $f(x) = \frac{1}{3}(x+2)^3 - 8$, occur based on its parent function? Parent function $y = x^3$
Horizontal shift left 2, vertical shift down 8, vertical shrink by $\frac{1}{3}$

12. Graph $f(x) = \frac{1}{2}x^2$,

x	y
-2	2
-1	1/2
0	0
1	1/2
2	2



$f(x) = \sqrt[3]{x-1}$



x	y
9	2
2	1
1	0
0	-1
-7	-2

13. Divide and express the result in standard form $\frac{3-4i}{3+4i} \left(\frac{3-4i}{3-4i} \right) \cdot \frac{-1}{-1}$

$a+bi$

$\frac{9-12i-12i+16i^2}{9-12i+12i-16i^2} = \frac{9-24i-16}{9+16} = \frac{-7-24i}{25}$

$\frac{-7}{25} - \frac{24i}{25}$

14. State whether the function crosses or turns around at each x-intercept $f(x) = x^3 + 6x^2 + 9x$

$$x^3 + 6x^2 + 9x$$

$$x(x^2 + 6x + 9)$$

$$x(x+3)(x+3)$$

X-int
 $(0, 0)$ & $(-3, 0)$
 ↑
 multiplicity of 1 so odd
 ↑
 multiplicity of 2 so even

When
 $x=0$ it crosses
 $x=-3$ it touches & turns.

15. Divide and find the remainder $f(x) = \frac{3x^4 + 2x^2 - 8x + 3}{x+3}$ ← missing $0x^3$

$$\begin{array}{r} -3 \overline{) 3 \ 0 \ 2 \ -8 \ 3} \\ + 0 \ -9 \ 27 \ -87 \ 285 \\ \hline 3 \ -9 \ 29 \ -95 \ 288 \end{array}$$

$$\text{or } 3(-3)^4 + 2(-3)^2 - 8(-3) + 3 = 288$$

Remainder: 288

16. Use Descartes's Rule of Signs to determine the amount of possible positive and negative zeros for

$$f(x) = 2x^3 + x^2 - 3x - 1$$

Poss. Zeros
 $2x^3 + x^2 - 3x - 1$
 1

1 possible positive real zero

Neg. Zeros

$$2(-1)^3 + (-1)^2 - 3(-1) - 1$$

$$-2x^3 + x^2 + 3x - 1$$

2 or possible ~~pos~~ negative real zeros

17. Find the vertical asymptote(s) of $f(x) = \frac{x-4}{x^2-x-6}$

V. Asym: $x = -2$
 $x = 3$

$$(x+2)(x-3)$$

18. Solve the rational inequality $f(x) = \frac{x^2+x-6}{x+1} > 0$ $(x+3)(x-2)$

Boundary Points

$$x = -3, 2, -1$$

test →
 PTS

$$(-\infty, -3) \quad (-3, -1) \quad (-1, 2) \quad (2, \infty)$$

$$-4 \quad -2 \quad 0 \quad 3$$

plug in see which are true & which are false

-4 False
 -2 true
 0 False
 3 true

Answer
 $(-3, -1) \cup (2, \infty)$

19. State the correct value for $\tan\left(-\frac{\pi}{2}\right)$, $\sin \pi$, $\cos \frac{13\pi}{6}$, $\sec \frac{\pi}{4}$, $\csc \frac{5\pi}{6}$, $\cot(-765^\circ)$, etc.

$$\tan\left(-\frac{\pi}{2}\right) = \tan\left(\frac{3\pi}{2}\right)$$

UND

$$\sin \pi = 0$$

$$\cos \frac{13\pi}{6} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sec \frac{\pi}{4} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\csc \frac{5\pi}{6} = \frac{1}{\frac{1}{2}} = 2$$

$$\cot(-765^\circ) = -1$$

★ USE THE UNIT CIRCLE

20. Use the Pythagorean Identity to find $\sin \theta$, given $\cos \theta = -\frac{4}{5}$ and $\pi < \theta < \frac{3\pi}{2}$

$$\cos \theta = \frac{x}{r} \quad \text{Q III}$$

$$x = -4 \quad y = ? \quad r = 5$$

$$(-4)^2 + y^2 = 5^2$$

$$16 + y^2 = 25$$

$$y^2 = 9$$

$$y = -3 \leftarrow \text{in Q III}$$

$$\sin \theta = \frac{y}{r}$$

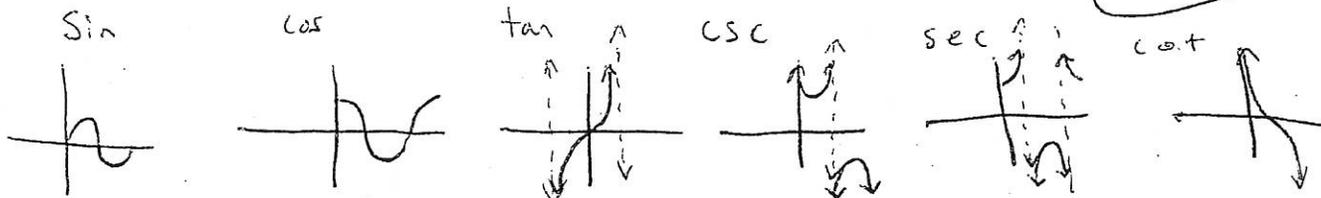
$$\sin \theta = \frac{-3}{5}$$

21. Find the exact value of the expression $\left(\cos \frac{3\pi}{4}\right)\cos\left(-\frac{\pi}{6}\right) + \left(\cos \frac{5\pi}{3}\right)\sin\frac{\pi}{2}$

$$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)(1) = -\frac{\sqrt{6}}{4} + \frac{1}{2}\left(\frac{2}{2}\right)$$

$$\frac{-\sqrt{6}+2}{4}$$

22. Be able to identify the graphs of each of the six trigonometric functions.



23. Determine the period of $y = \frac{2}{3}\cos\left(\frac{2}{5}x - \frac{\pi}{4}\right)$

Period $\frac{2\pi}{B}$ $\frac{2\pi}{2/5} = 2\pi \cdot \frac{5}{2} = 5\pi$

24. Determine whether the following is True or False:

$$\frac{\sin x}{\cos x + 1} + \frac{\cos x - 1}{\sin x} = 0$$

test a value like 0.

$$\frac{\sin 0}{\cos 0 + 1} + \frac{\cos 0 - 1}{\sin 0} = 0?$$

$$\frac{0}{1+1} + \frac{2}{0} = 0$$

False

25. Determine whether the following is True or False:

$$\frac{\cos(\alpha - \beta)}{\cos \alpha \sin \beta} = \tan \alpha + \cot \beta$$

$$\frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\cos \alpha \sin \beta}$$

$$\frac{\cancel{\cos \alpha} \cos \beta}{\cancel{\cos \alpha} \sin \beta} + \frac{\sin \alpha \cancel{\sin \beta}}{\cancel{\cos \alpha} \sin \beta}$$

$$\frac{\cos \beta}{\sin \beta} + \frac{\sin \alpha}{\cos \alpha}$$

$$\cot \beta + \tan \alpha$$

$$\tan \alpha + \cot \beta$$

26. Use the identities for $\cos(x \pm y)$ to evaluate $\cos 105^\circ$

$$105^\circ = 60^\circ + 45^\circ$$

$$\cos(60^\circ + 45^\circ)$$

$$\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

27. Find the exact value using identities for $\tan(x \pm y)$: $\frac{\tan 25^\circ + \tan 35^\circ}{1 - \tan 25^\circ \tan 35^\circ}$ ← Same as $\frac{\tan \alpha + \tan \beta}{1 + \tan \alpha \tan \beta} = \tan(\alpha + \beta)$

So $\tan(25^\circ + 35^\circ)$
 $\tan 60^\circ = \sqrt{3}$

28. Find the exact value of $\cos(\alpha + \beta)$, if $\cos \alpha = \frac{5}{17}$, α lies in quadrant I, and $\sin \beta = -\frac{3}{4}$, β lies in quadrant IV

Need to find $\cos \beta$ & $\sin \alpha$

$$\cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\left(\frac{5}{17}\right)\left(\frac{\sqrt{7}}{4}\right) - \left(\frac{2\sqrt{66}}{17}\right)\left(-\frac{3}{4}\right)$$

$$\frac{5\sqrt{7}}{68} - \frac{(2\sqrt{66})(-3)}{68} = \frac{5\sqrt{7} + 6\sqrt{66}}{68}$$

$\cos \alpha = \frac{5}{17}$ $5^2 + y^2 = 17^2$
 $y^2 = 264$
 $y = \sqrt{264} = 2\sqrt{66}$
 $\sin \alpha = \frac{2\sqrt{66}}{17}$

$\sin \beta = -\frac{3}{4}$ Q IV
 $(-3)^2 + x^2 = 4^2$
 $9 + x^2 = 16$
 $x^2 = 7$
 $x = \sqrt{7}$

29. Find the exact value of $\sin 2\theta$, if $\sin \theta = \frac{15}{17}$, θ lies in quadrant II

$2 \sin \theta \cos \theta$

$$2\left(\frac{15}{17}\right)\left(\frac{8}{17}\right) = \frac{240}{289}$$

$x^2 + 15^2 = 17^2$
 $x^2 = 64$
 $x = 8$

30. Solve for $0 \leq x \leq 2\pi$, $\sin x = -\frac{\sqrt{2}}{2}$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}$

31. Solve: $2\cos^2 x + 3\cos x + 1 = 0$ for $0 \leq x < 2\pi$

$(2\cos x + 1)(\cos x + 1) = 0$

$\frac{2\cos x + 1}{2} = \frac{-1}{2}$ $\cos x = -1$
 $\cos x = -\frac{1}{2}$ $x = \pi$

$2\pi/3, 4\pi/3$