PC Final Exam Review Answers

(1) $x + 5y = 8$

$r\cos\theta + 5r\sin\theta = 8$

$r(\cos\theta + 5\sin\theta) = 8$

$\boxed{r = \dfrac{8}{\cos\theta + 5\sin\theta}}$

(2) $z_1 z_2 =$

$4(7)\left(\cos(15+25)^\circ + i\sin(15+25)^\circ\right)$

$= \boxed{28\left(\cos 40^\circ + i\sin 40^\circ\right)}$

(3) $A = P\left(1+\frac{r}{n}\right)^{nt}$

semi-annually $n=2$

$A = 5{,}000\left(1+\frac{0.065}{2}\right)^{2(6)}$

\$7,339.23

quarterly $n=4$

$A = 5{,}000\left(1+\frac{0.065}{4}\right)^{4(6)}$

\$7,361.79

(4) (a) $\log_4 \frac{1}{4} = \log_4 4^{-1} = \boxed{-1}$

(b) $\ln e^{3/x} = \boxed{3/x}$

(5) (a) $8^{1-x} = 4^{x+2}$

$2^{3(1-x)} = 2^{2(x+2)}$

$3 - 3x = 2x + 4$

$-5x = 1$

$\boxed{x = -\frac{1}{5}}$

(b) $9^x = 3^{-1/3}$

$3^{2x} = 3^{-1/3}$

$2x = -1/3$

$\boxed{x = -1/6}$

(c) $\log_2(x+2) - \log_2(x-5) = 3$

$\dfrac{(x+2)}{(x-5)} = 2^3$

$x+2 = 8(x-5)$

$x+2 = 8x - 40$

$-7x = -42$

$\boxed{x = 6}$

(d) $\ln(x-4) + \ln(x+1) = \ln(x-8)$

$(x-4)(x+1) = x-8$

$x^2 - 3x - 4 = x - 8$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$\boxed{x = 2}$

(6) $70 = 100\left(\frac{1}{2}\right)^{t/12}$

$0.70 = \left(\frac{1}{2}\right)^{t/12}$

$\ln 0.70 = \frac{t}{12}\ln 0.5$

$\frac{12 \ln 0.70}{\ln 0.5} = t$

$\boxed{t = 6 \text{ hours}}$

(7) $\begin{cases} xy = 3 \\ x^2+y^2=10 \end{cases}$

$y = \frac{3}{x}$

$\left(\frac{9}{x^2} + x^2\right)x^2 = (10)x^2$

$9 + x^4 = 10x^2$

$x^4 - 10x^2 + 9 = 0$

$(x^2-9)(x^2-1)$

$(x+3)(x-3)(x+1)(x-1)$

$-3 \quad 3 \quad -1 \quad 1$

$\boxed{\begin{matrix}(-3,-1)\\(3,1)\\(-1,-3)\\(1,3)\end{matrix}}$

(7b) $\begin{cases} 3x^2-2y^2=1 \\ 4x-y=3 \end{cases} \rightarrow y = 4x-3$

$3x^2 - 2(4x-3)^2 = 1$

$3x^2 - 2(16x^2-24x+9) = 1$

$3x^2 - 32x^2 + 48x - 18 = 1$

$-29x^2 + 48x - 19 = 0$

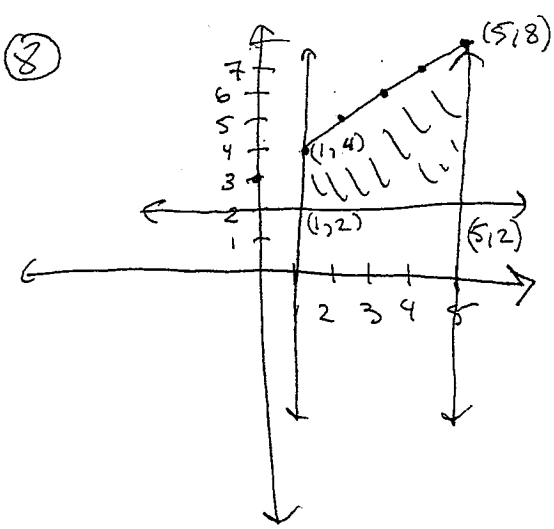
$29x^2 - 48x + 19 = 0$

$(x - 1)(29x - 19)$

$1 \quad 19/29$

$\boxed{\begin{matrix}(1,1)\\\left(\frac{19}{29}, -\frac{11}{29}\right)\end{matrix}}$

(8)



$x - y \geq -3$

$x + 3 \geq y$

$y \leq x + 3$

Find edge points

$(1,4)$ min

$(1,2)$

$(5,2)$ max

$(5,8)$

$z = 3x - 2y$

$z = 3(1) - 2(4) = -5$ min

$z = 3(1) - 2(2) = -1$

$z = 3(5) - 2(2) = 11$ max

$z = 3(5) - 2(8) = -1$

(9) regular $\to r$ deluxe $\to d$

$$\begin{cases} r \geq 50 \\ d \geq 75 \\ r+d \leq 150 \end{cases} \qquad P = 200r + 250d$$

(10)
$$\left[\begin{array}{ccc|c} 2 & 2 & 7 & -1 \\ 2 & 1 & 2 & 2 \\ 4 & 6 & 1 & 15 \end{array}\right] \xrightarrow[R_2]{R_2 - R_1} \left[\begin{array}{ccc|c} 2 & 2 & 7 & -1 \\ 0 & -1 & -5 & 3 \\ 4 & 6 & 1 & 15 \end{array}\right]$$

$$\xrightarrow[R_3]{R_3 - 2R_2} \left[\begin{array}{ccc|c} 2 & 2 & 7 & -1 \\ 0 & -1 & -5 & 3 \\ 0 & 4 & -3 & 11 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 1 & 7/2 & -1/2 \\ 0 & -1 & -5 & 8 \\ 0 & 4 & -3 & 11 \end{array}\right]$$

$$\xrightarrow[R_3 + 4R_2 \to R_3]{R_1 + R_2 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1.5 & 2.5 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & -23 & 23 \end{array}\right] \xrightarrow{R_3/-23} \left[\begin{array}{ccc|c} 1 & 0 & -1.5 & 2.5 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow[R_1 + 1.5R_2 \to R_1]{R_2 - 5R_3 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\boxed{\text{Answer: } (1, 2, -1)}$$

(11)
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 2-4 & 1+6 & 4-2 \\ 4-8 & 2+12 & 8-4 \\ 6+2 & 3-3 & 12+1 \end{bmatrix} = \begin{bmatrix} -2 & 7 & 2 \\ -4 & 14 & 4 \\ 8 & 0 & 13 \end{bmatrix}$$

(12)
$$\det \begin{bmatrix} 7 & 9 \\ -2 & -5 \end{bmatrix} = 7(-5) - (-2)(9) = -35 + 18 = \boxed{-17}$$

(13) see next page (14) yes draw picture

(15) (a) ellipse (b) hyperbola

(16)

(a) $\lim_{x \to 3} \frac{1}{x-2} = \frac{1}{3-2} = 1$

(b) $\lim_{x \to -5} \frac{x^2 - 25}{x - 4} = \frac{0}{-9} = 0$

(c) $\lim_{x \to -5} \frac{(x^2 - 25)}{x - 5} = \lim_{x \to -5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}} = -5 + 5 = 0$

(13)

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -4 & 0 & 1 & 0 \\ 2 & 4 & -4 & 1 & 0 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\to R_3]{R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -4 & 0 & 1 & 0 \\ 2 & 4 & -4 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 \end{array}\right]$$

$$\xrightarrow{R_3/-1} \left[\begin{array}{ccc|ccc} 1 & 3 & -4 & 0 & 1 & 0 \\ 2 & 4 & -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow[R_1]{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & -1 & 0 \\ 2 & 4 & -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2/2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & -1 & 0 \\ 1 & 2 & -2 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow[R_2]{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & -1 & 0 \\ 0 & -1 & 2 & 0.5 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[\text{First } R_1 + R_2 \to R_1]{\text{second } R_2/-1} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1.5 & -2 & 0 \\ 0 & 1 & -2 & -0.5 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 3.5 & -2 & -2 \\ -2.5 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[R_2 + 2R_3 \to R_2]{R_1 - 2R_3 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3.5 & -2 & -2 \\ 0 & 1 & 0 & -2.5 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

(17)

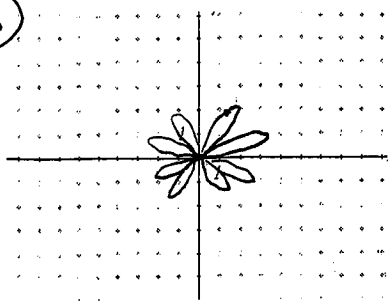
$$\lim_{h \to 0} \frac{(x+h)^2 - 4(x+h) + 7 - x^2 + 4x - 7}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{4x} - 4h + \cancel{7} - \cancel{x^2} + \cancel{4x} - \cancel{7}}{h}$$

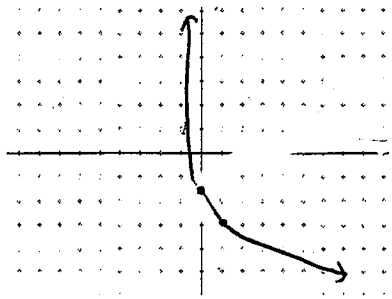
$$= \lim_{h \to 0} \frac{h(h + 2x - 4)}{h} = \lim_{h \to 0} h + 2x - 4 = 2x - 4$$

So $f'(x) = 2x - 4$ and $f'(2) = 0$

18 Graph $r = 3\sin 4\theta$

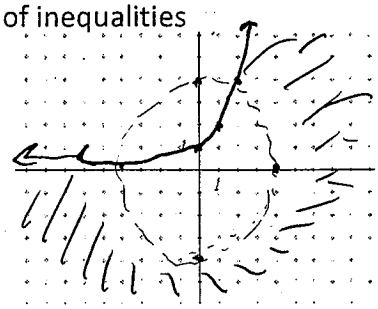


19 Graph $f(x) = -\log_2(x+1) - 2$.



20 Graph the system of inequalities

A $\begin{cases} x^2 + y^2 > 16 \\ y \le 2^x \end{cases}$

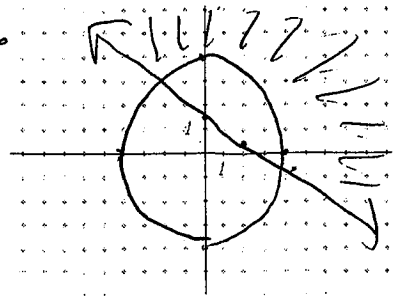


B $\begin{cases} x^2 + y^2 \ge 16 \\ 2x + 4y \ge 6 \end{cases}$ $0 \ge 16$ $0 \ge 6$

$x + 2y \ge 3$

$2y \ge -x + 3$

$y \ge -\frac{x}{2} + \frac{3}{2}$



Graph each of the following conic sections. Identify all key characteristics such as center, vertex, focus(foci), directrix, major axis, minor axis, and equations of asymptotes where applicable.

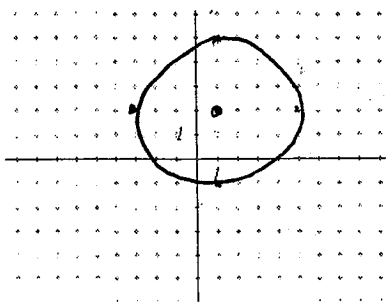
21 a. $\frac{(x-1)^2}{16} + \frac{(y-2)^2}{9} = 1$

center $(1, 2)$

major axis: 8

minor axis: 6

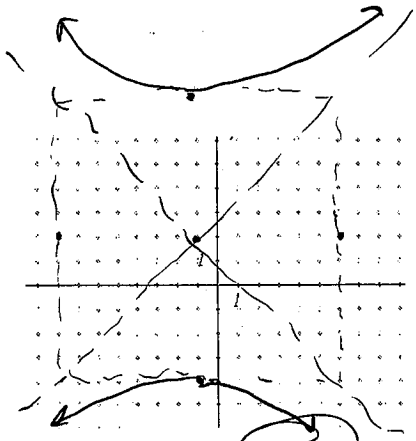
foci $(1 \pm \sqrt{7}, 2)$



22 b. $\frac{(y-2)^2}{36} - \frac{(x+1)^2}{49} = 1$

center $(-1, 2)$

foci $(-1, 2 \pm \sqrt{85})$



23 c. $(y+3)^2 = 12(x+1)$

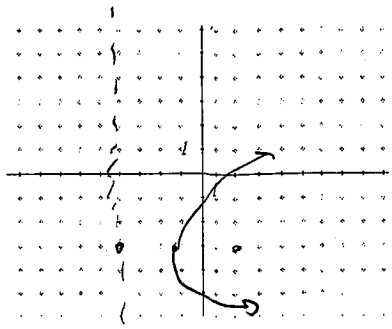
$4f = 12$

$f = 3$

center $(-1, -3)$

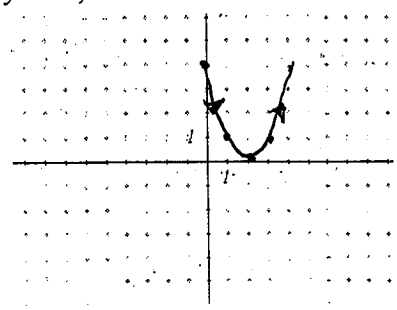
directrix $x = -4$

focus $(2, -3)$

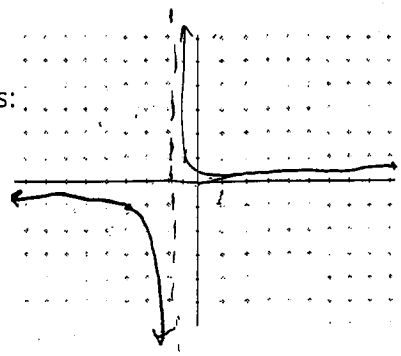


24 Graph the plane curve described by the parametric equations $x = t + 2$, $y = t^2$, for $-2 \le t \le 2$.

t	x	y
-2	0	4
-1	1	1
0	2	0
1	3	1
2	4	4



25 Graph $f(x) = \frac{1}{x+2}$.



Find the following limits:

a. $\lim_{x \to -2^-} f(x) = -\infty$

b. $\lim_{x \to -2^+} f(x) = +\infty$

c. $\lim_{x \to -2} f(x)$ DNE