

Trigonometry Practice Review

Name Key Hour: _____

$$x = -5 \quad y = -12 \quad r = 13 \quad (-5)^2 + (12)^2 = r^2 \\ r = 13$$

If $\tan\theta = \frac{12}{5}$ when $\pi < \theta < \frac{3\pi}{2}$, set up and solve for the following

1. $\sin\theta =$

$$\frac{-12}{13}$$

Q III

$$\sin\theta = \frac{-12}{13}$$

$$\cos\theta = \frac{-5}{13}$$

2. $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

$$\frac{1-(-5/13)}{-12/13} = \frac{\frac{13}{13} + \frac{5}{13}}{\frac{-12}{13}} = \frac{\frac{18}{13}}{\frac{-12}{13}} = \frac{18}{-12} = \frac{3}{-2}$$

$$\tan\frac{\theta}{2} = \frac{3}{-2}$$

2. $\sin\frac{\theta}{2} = \pm \sqrt{\frac{1-\cos\theta}{2}}$

$$\pm \sqrt{\frac{1+(-5/13)}{2}} = \sqrt{\frac{\frac{13}{13} + \frac{5}{13}}{2}} = \sqrt{\frac{\frac{18}{13}}{2}}$$

$$\pm \sqrt{\frac{18}{13} \cdot \frac{1}{2}} = \pm \sqrt{\frac{9}{13}} = \pm \frac{\sqrt{9}}{\sqrt{13}} = \pm \frac{3}{\sqrt{13}} = \pm \frac{3}{\sqrt{13}} \left(\frac{\sqrt{13}}{\sqrt{13}} \right)$$

* If θ is in Q III
then $\theta/2$ is in Q II

$$\sin\frac{\theta}{2} = \frac{3\sqrt{13}}{13}$$

4. $\cos 2\theta$

$$\cos^2\theta - \sin^2\theta$$

$$\left(\frac{-5}{13}\right)^2 - \left(\frac{-12}{13}\right)^2$$

$$\frac{25}{169} - \frac{144}{169} = \boxed{\frac{-119}{169}}$$

5. Simplify

A) $\cos^2(21^\circ) - \sin^2(21^\circ)$

$$\cos 2(21) = \boxed{\cos 42^\circ}$$

B) $\frac{\tan 3^\circ + \tan 9^\circ}{1 - \tan 3^\circ \tan 9^\circ}$

$$\tan(3+9) = \boxed{\tan 12^\circ}$$

C) $2\sin 76^\circ \cos 76^\circ$

$$\sin 2(76) = \boxed{\sin 152^\circ}$$

D) $\sin 16^\circ \cos 17^\circ + \cos 16^\circ \sin 17^\circ$

$$\sin(16+17) = \boxed{\sin 33^\circ}$$

E) $2\cos^2(13^\circ) - 1$

$$\cos 2(13) = \boxed{\cos 26^\circ}$$

F) $\frac{\sin 33^\circ}{1 + \cos 33^\circ}$

$$\tan\frac{\theta}{2} = \tan\frac{33^\circ}{2}$$

or

$$\tan 16.5^\circ$$

6. Express the following as a function of γ alone angles whose functional values $\cos(\pi + \gamma)$

$$\begin{aligned} & \cos \pi \cos \gamma - \sin \pi \sin \gamma \\ \rightarrow & \cos \gamma - 0 \sin \gamma \\ & \boxed{-\cos \gamma} \end{aligned}$$

7. Write the expression $\sin \frac{7\pi}{12}$ as the sum of known angles and then evaluate

$$\begin{aligned} \sin \frac{7\pi}{12} &= \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin \frac{\pi}{4} \cos \frac{\pi}{3} + \cos \frac{\pi}{4} \sin \frac{\pi}{3} \\ \frac{3\pi}{12} + \frac{4\pi}{12} &= \frac{7\pi}{12} \quad \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}} \end{aligned}$$

8. Verify: $\cos x \cot x = \frac{1 - \sin^2 x}{\sin x}$

$$\begin{aligned} \cos x &= \frac{\cos x}{\sin x} \\ \frac{\cos^2 x}{\sin x} &= \frac{1 - \sin^2 x}{\sin x} \end{aligned}$$

9. Verify: $(\tan^2 \theta + 1)(\sec^2 \theta + 1) = \tan^2 \theta + 2$

$$\begin{aligned} & \sec^2 \theta (\sec^2 \theta + 1) \\ & \sec^2 \theta \sec^2 \theta + \sec^2 \theta \\ & \cancel{\frac{1}{\sec^2 \theta}} \cdot \cancel{\sec^2 \theta} + \sec^2 \theta \\ & 1 + \sec^2 \theta \end{aligned} \quad \begin{aligned} & 1 + (1 + \tan^2 \theta) \\ & 2 + \tan^2 \theta \\ & \boxed{\tan^2 \theta + 2} \end{aligned}$$

10. Simplify $\frac{\tan \frac{\pi}{12} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{12} \tan \frac{\pi}{4}} = \tan\left(\frac{\pi}{12} + \frac{\pi}{4}\right) = \tan\left(\frac{\pi}{12} + \frac{3\pi}{12}\right) = \tan\left(\frac{4\pi}{12}\right) = \tan \frac{\pi}{3}$

$$\tan \frac{\pi}{3} = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \cancel{\frac{1}{1}} = \boxed{\sqrt{3}}$$

11. Verify $\frac{\cos(\alpha - \beta)}{\sin \alpha \sin \beta} = \cot \alpha \cot \beta + 1$

$$\frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\sin \alpha \sin \beta}$$

$$\frac{\cos \alpha \cos \beta}{\sin \alpha \sin \beta} + \frac{\sin \alpha \sin \beta}{\sin \alpha \sin \beta}$$

$$\cot \alpha \cot \beta + 1$$