

Warm-Up

$$y = r \sin \theta$$

$$x = r \cos \theta$$

$$r = \sqrt{x^2 + y^2} \quad \text{or} \quad r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$

Convert to rectangular form.

$$1) r = (-4 \sin \theta) r$$

$$r^2 = 4r \sin \theta$$

$$x^2 + y^2 = -4y$$

$$\begin{aligned} & x^2 + y^2 + 4y + 4 = 0 + 4 \\ & x^2 + (y+2)^2 = 4 \end{aligned}$$

$$\left(\frac{4}{2}\right)^2 = 2^2 = 4$$

Convert to polar form.

$$2) x^2 = 6y$$

$$\frac{r^2 \cos^2 \theta}{r} = \frac{6r \sin \theta}{r}$$

$$\frac{r \cos^2 \theta}{\cos^2 \theta} = \frac{6 \sin \theta}{\cos^2 \theta}$$

$$r = \frac{6 \sin \theta}{\cos^2 \theta}$$

Plotting Complex #'s

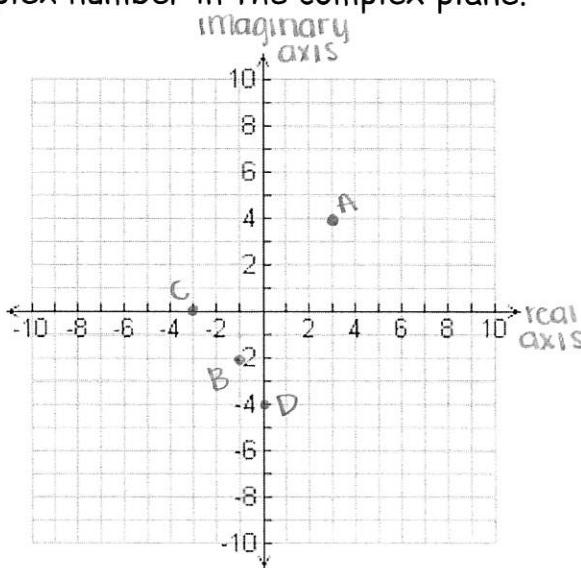
ex) Plot each complex number in the complex plane.

$$a) z = 3 + 4i$$

$$b) z = -1 - 2i$$

$$c) z = -3 + 0i$$

$$d) z = -4i$$



complex numbers
 $z = a + bi$ or $z = x + yi$

Absolute Value of a Complex Number

$$|z| = |a+bi| = \sqrt{a^2 + b^2}$$

Ex2) Find the absolute value of each.

a) $z = 3+4i$

$$\begin{aligned} &\sqrt{3^2 + 4^2} \\ &\sqrt{9+16} \\ &\sqrt{25} = 5 \end{aligned}$$

b) $z = -1-2i$

$$\begin{aligned} &\sqrt{(-1)^2 + (-2)^2} \\ &\sqrt{1+4} \\ &\sqrt{5} \end{aligned}$$

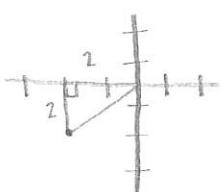
Polar Form of a Complex Number

rectangular \rightarrow polar
 $z = a+bi$ in polar is $z = r(\cos \theta + i \sin \theta)$

where $a = r \cos \theta$, $b = r \sin \theta$, $r = \sqrt{a^2 + b^2}$ and $\tan \theta = \frac{b}{a}$

ex3) Graph $z = -2-2i$ then find the polar form of z .

Q III



$$\tan \theta = \frac{-2}{-2} = 1$$

$$\theta = \tan^{-1}(1) = \frac{\pi}{4}$$

$$\theta = \frac{5\pi}{4}$$

$$r = \sqrt{(-2)^2 + (-2)^2}$$

$$\sqrt{4+4}$$

$$\sqrt{8}$$

$$r = 2\sqrt{2}$$

Ex4) Write $z = 2(\cos 60^\circ + i \sin 60^\circ)$ in rectangular form

$$z = 2(\cos 60^\circ + i \sin 60^\circ)$$

$$2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$z = 1 + i\sqrt{3}$$

$$z = a+bi$$

$$z = 2\sqrt{2} \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right)$$

Product of Two complex numbers in polar form.

$$z_1 = r_1(\cos \theta_1 + i \sin \theta_1) \quad \text{and} \quad z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$$

$$z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$$

ex5) Find the product of z_1 and z_2 , leave in polar form.

$$z_1 = 4(\cos 50^\circ + i \sin 50^\circ) \quad z_2 = 7(\cos 100^\circ + i \sin 100^\circ)$$

$$z_1 z_2 = 28 (\cos 150^\circ + i \sin 150^\circ)$$

Quotient of Two complex numbers in polar form.

$$z_1 = r_1(\cos \theta_1 + i \sin \theta_1) \quad \text{and} \quad z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$$

ex6) Find the quotient of $\frac{z_1}{z_2}$, leave in polar form.

$$z_1 = 12(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}) \quad z_2 = 4(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$$

$$\frac{z_1}{z_2} = \frac{12}{4} \left[\cos \left(\frac{3\pi}{4} - \frac{\pi}{4} \right) + i \sin \left(\frac{3\pi}{4} - \frac{\pi}{4} \right) \right]$$

$$\frac{z_1}{z_2} = 3 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

DeMoivre's Theorem

$$z^n = [r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

EX7) Find $z^6 = [2(\cos 20^\circ + i \sin 20^\circ)]^6$ then write in rectangular form

$$2^6 [\cos(6 \cdot 20^\circ) + i \sin(6 \cdot 20^\circ)]$$

$$64(\cos 120^\circ + i \sin 120^\circ) \leftarrow \text{polar}$$

$$64\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$\boxed{z^6 = -32 + 32i\sqrt{3}}$$

EX8) Find $z^8 = (1+i)^8$ using DeMoivre's Theorem, then write in rectangular form

$$r = \sqrt{x^2 + y^2}$$

$$\sqrt{1^2 + 1^2}$$

$$\sqrt{1+1}$$

$$\sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

$$z^8 = \left[\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]^8$$

$$\sqrt{2}^8 [\cos(8 \cdot \frac{\pi}{4}) + i \sin(8 \cdot \frac{\pi}{4})]$$

$$16(\cos 2\pi + i \sin 2\pi) \leftarrow \text{polar}$$

$$16[1+i(0)]$$

$$\boxed{z^8 = 16} \quad \text{or} \quad \boxed{z^8 = 16 + 0i}$$