Pre-Calc Notes: 4.2 Trig. Function: Unit Circle Friday 10/21/16

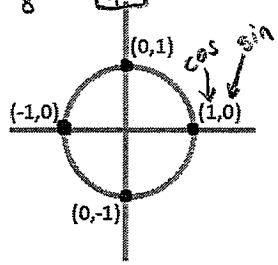
Warm-Up — get the root to the numerator.

Rationalize the following

1) $\frac{3}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{3\sqrt{5}}{5}$ 2) $\frac{7}{3\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{7\sqrt{5}}{3\cdot 5} = \frac{7\sqrt{5}}{15}$ 3) $\frac{2}{\sqrt{8}}\left(\frac{\sqrt{8}}{\sqrt{8}}\right) = \frac{2\sqrt{8}}{8} = \frac{2\cdot\sqrt{4}\sqrt{2}}{8} = \frac{2\cdot 2\sqrt{2}}{8} = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$ 4) $\frac{-1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{-\sqrt{3}}{3}$

Six Trig. Functions

$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$y \neq 0$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$x \neq 0$
$\tan\theta = \frac{y}{x}$ $x\neq 0$	$\cot\theta = \frac{x}{y}$	$y \neq 0$



*(cos,sin)

**If you are using the unit circle, the radius is always 1.

ie.) $\sin\theta = \frac{y}{1}$ or $\sin\theta = y$

Ex1) θ is a real number equal to the length of the intercepted arc of an angle that measures θ radians and $P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is a point on the unit circle that corresponds to θ. (x = cos, y = sin)

Find the value of the six trig functions.

$\sin\theta = \sqrt{3}/2$ $\quad \csc\theta = \frac{2\sqrt{3}}{3}$

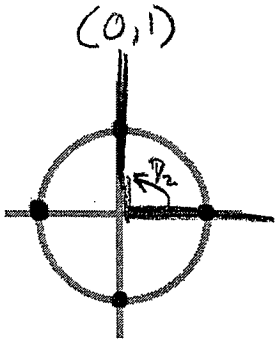
$\cos\theta = -\frac{1}{2}$ $\quad \sec\theta = -2$

$\tan\theta = -\sqrt{3}$ $\quad \cot\theta = -\frac{\sqrt{3}}{3}$

$\tan\theta = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{\sqrt{3}}{2}\cdot\frac{-2}{1} = -\sqrt{3}$ $\quad \csc\theta = \frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{2\sqrt{3}}{3}$

$\cot\theta = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}} = \frac{-1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = -\frac{\sqrt{3}}{3}$

Ex2) Find the value of each trig function at $\theta = \frac{\pi}{2}$.



$\tan\theta = \frac{1}{0}$

$\sec\theta = \frac{1}{0}$

$\cot\theta = \frac{0}{1}$

$\sin\theta = 1$

$\cos\theta = 0$

$\tan\theta =$ UND

$\csc\theta = 1$

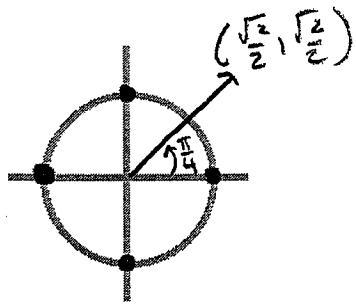
$\sec\theta =$ UND

$\cot\theta = 0$

Ex3) Find the value of each of the following.

$\sin\frac{\pi}{4} =$ $\sqrt{2}/2$ $\quad \cos\frac{\pi}{4} =$ $\sqrt{2}/2$ $\quad \tan\frac{\pi}{4} =$ 1

$\frac{\sqrt{2}/2}{\sqrt{2}/2}$



Even and Odd Trig Functions

*The cosine and secant functions are "Even".

$\cos(-\theta) = \cos\theta$ $\qquad$ $\sec(-\theta) = \sec\theta$

*The sine, cosecant, tangent, and cotangent functions are "Odd".

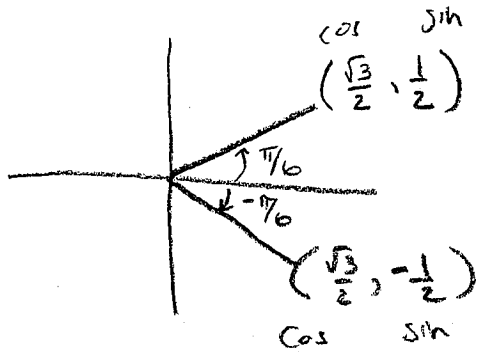
$\sin(-\theta) = -\cos\theta$ $\qquad$ $\csc(-\theta) = -\csc\theta$

$\tan(-\theta) = -\tan\theta$ $\qquad$ $\cot(-\theta) = -\tan\theta$

Ex4) Use the unit circle to understand odd vs even functions.

$\cos\frac{\pi}{6} =$ $\sqrt{3}/2$ $\qquad$ $\sin\frac{\pi}{6} =$ $1/2$

$\cos(-\frac{\pi}{6}) =$ $\sqrt{3}/2$ $\qquad$ $\sin(-\frac{\pi}{6}) =$ $-1/2$



Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Ex5) Given $sin\theta = \frac{2}{5}$ and $cos\theta = \frac{\sqrt{21}}{5}$, find the value of the four remaining trig. functions.

What quadrant would this happen in? QI

$\tan\theta = \frac{\frac{2}{5}}{\frac{\sqrt{21}}{5}} = \frac{2}{5} \cdot \frac{5}{\sqrt{21}} = \frac{2}{\sqrt{21}}\left(\frac{\sqrt{21}}{\sqrt{21}}\right) = \frac{2\sqrt{21}}{21}$

$\sec\theta = \frac{5}{\sqrt{21}}\left(\frac{\sqrt{21}}{\sqrt{21}}\right) = \frac{5\sqrt{21}}{21}$

$\cot\theta = \frac{\frac{\sqrt{21}}{5}}{\frac{2}{5}} = \frac{\sqrt{21}}{5} \cdot \frac{5}{2} = \frac{\sqrt{21}}{2}$

$\sin\theta = \frac{2/5}{}$

$\cos\theta = \frac{\sqrt{21}/5}{}$

$\tan\theta = \frac{2\sqrt{21}}{21}$

$\csc\theta = \frac{5/2}{}$

$\sec\theta = \frac{5\sqrt{21}}{21}$

$\cot\theta = \frac{\sqrt{21}}{2}$

Pythagorean Identities

$\sin^2\theta + cos^2\theta = 1$

There are two other Pythagorean Identities that you can derive from this

Divide everything by $\sin^2\theta$

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$1 + \cot^2\theta = \csc^2\theta$

Divide everything by $cos^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

Ex6) Given that $\sin\theta = \frac{3}{5}$ and $0 \le \theta \le \frac{\pi}{2}$, find the value of $\cos\theta$ using a trig. identity.

What does $0 \le \theta \le \frac{\pi}{2}$ tell you?? QI

$\sin^2\theta + \cos^2\theta = 1$

$(3/5)^2 + \cos^2\theta = 1$

$\frac{9}{25} + \cos^2\theta = 1$

$-\frac{9}{25}$ $\quad -\frac{9}{25}$

$\cos^2\theta = 1 - \frac{9}{25}$

$\cos^2\theta = \frac{25}{25} - \frac{9}{25}$

$\cos^2\theta = \frac{16}{25}$

$\cos\theta = \frac{4}{5}$

Ex7) Find the value of each trig function.

a) $\sin\frac{9\pi}{4} =$ $\sqrt{2}/2$

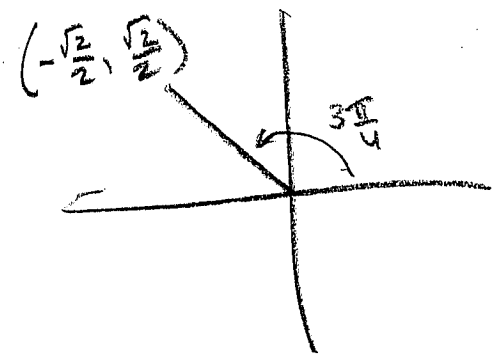
Coterminal

$\frac{9\pi}{4} - \frac{8\pi}{4} = \frac{\pi}{4}$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

b) $\tan\left(-\frac{5\pi}{4}\right)$

Coterminal

$-\frac{5\pi}{4} + \frac{8\pi}{4} = \frac{3\pi}{4}$

$\tan\frac{3\pi}{4} = -1$