10.3 Geometric Sequences & Series

A **geometric sequence** is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed nonzero constant. The amount we multiply each time is called the **common ratio**, "r".

Ex1 Write the first six terms of the geometric sequence with the first term as 6 & the common ratio is 1/3.

Solution

$$6,\ 2,\ \frac{2}{3},\ \frac{2}{9},\ \frac{2}{27}\ \&\ \frac{2}{81}$$

$\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$

General Term of a Geometric Sequence

$$\boxed{a_n = a_1 r^{n-1}}$$

* If you know the first term and the common ratio, then you can find any term you may need!

Ex2 Find the eighth term of the geometric sequence whose first term is -4 and whose common ratio is -2.

Solution

$a_n = a_1 r^{n-1}$ $\quad a_8 = -4(-2)^{8-1} = -4(-2)^7$

$-4(-128) = 512$

$A_8 = 512$

Ex3

Year	2000	2001	2002	2003	2004	2005
Population (millions)	281.4	284.5	287.6	290.8	294.0	297.2

A) Show that the pop. increases geometrically.

$$\frac{284.5}{281.4} \approx 1.01 \quad \frac{287.6}{284.5} \approx 1.01 \quad \frac{290.8}{287.6} \approx 1.01 \quad \frac{294.0}{290.8} \approx 1.01 \quad \frac{297.2}{294.0} \approx 1.01$$

All the same, the common ratio is 1.01

B) Write the general term of the geometric sequence for population, n years after 1999.

$$a_n = 281.4(1.011)^{n-1}$$

C) Project the population for 2018

$n = 20$

$$a_{20} = 281.4(1.011)^{20} = 343.36$$

Approx. 343.4 million in 2019

The Sum of the first n terms of a geometric sequence.

$$\boxed{S_n = \frac{a_1(1-r^n)}{1-r}}$$

$\boxed{\text{Ex4}}$ Find the sum of the first 18 terms of the geometric Sequence: $2, -8, 32, -128 \ldots$

* we need $r = \_\_\_$

$$\frac{-8}{2} = -4 \quad \frac{32}{-8} = -4 \quad \frac{-128}{32} = -4 \qquad r = -4$$

$$S_{18} = \frac{a_1(1-r^{18})}{1-r} = \frac{2(1-(-4)^{18})}{1-(-4)}$$

$$S_{18} = -27{,}487{,}790{,}694$$

$\boxed{\text{Ex5}}$ Find the following sum: $\sum_{i=1}^{10} 6 \cdot 2^i$

$a_1 = 6 \cdot 2^1 = 6 \cdot 2 = 12$
$a_2 = 6 \cdot 2^2 = 6 \cdot 4 = 24$
$a_3 = 6 \cdot 2^3 = 6 \cdot 8 = 48$

$$\frac{24}{12} = 2 \quad \frac{48}{24} = 2 \qquad r = 2$$

$$S_{10} = \frac{12(1-r^{10})}{1-r} = \frac{12(1-2^{10})}{1-2} = \frac{12(-1023)}{-1}$$

$$S_{10} = 12{,}276$$

The Sum of an infinite geometric series

$$\boxed{S = \frac{a_1}{1-r}}$$

$\boxed{\text{Ex 6}}$ Find the sum of the infinite geometric series:

$$\frac{3}{8} - \frac{3}{16} + \frac{3}{32} - \frac{3}{64} + \ldots$$

$$r = \frac{\frac{-3}{16}}{\frac{3}{8}} = -\tfrac{1}{2} \qquad S = \frac{3/8}{1-(-1/2)} = \frac{3/8}{3/2} = \frac{3}{8} \cdot \frac{2}{3} = \boxed{\frac{1}{4}}$$