

Use the following formulas when appropriate

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$a_n = a_1 r^{n-1}$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S = \frac{a_1}{1-r}$$

1: Given: 1, 5, 9, 13, ...

Determine the common difference, the nth term, and the 100th term of the arithmetic sequence.

Common diff: 4

$$n^{\text{th}} \text{ term: } a_n = 1 + (n-1) \cdot 4$$

100th term:

$$1 + 4n - 4$$

$$a_{100} = 4(100) - 3$$

$$a_n = 4n - 3$$

$$400 - 3$$

$$a_{100} = 397$$

2: The 12th term of an arithmetic sequence is 32, and the 5th term is 18. Find the common difference, the nth term, and the 20th term.

There are 7 terms from the 5th to the 12th. That means to go from a₅ to a₁₂, you need to a "d" seven times.

$$32 - 18 = 14$$

$$\frac{14}{7} = \frac{7d}{7}$$

$$d = 2$$

$$a_n = a_1 + (n-1)d$$

$$a_n = 10 + (n-1)2$$

We can solve for a₁, use a₅ = 18.

$$a_n = 10 + 2n - 2$$

$$18 = a_1 + (5-1)2$$

$$18 = a_1 + 4(2)$$

$$18 = a_1 + 8 \\ -8 \\ a_1 = 10$$

$$a_{20} = 2(20) + 8$$

$$a_{20} = 48$$

3: Given: 144, 12, 1, $\frac{1}{12}$,

Determine the common ratio, the fifth term, and the nth term of the geometry sequence.

$$\frac{12}{144} = \frac{1}{12}$$

$$a_4 \cdot r = a_5$$

$$\frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$r = \frac{1}{12}$$

$$a_n = 144 \left(\frac{1}{12}\right)^{n-1}$$

4: The common ratio in a geometric sequence is $\frac{2}{3}$, and the fourth term is $\frac{5}{2}$. Find the first three terms.

$$r = \frac{2}{3} \quad a_4 = \frac{5}{2}$$

Plug these in to

$$a_n = a_1 r^{n-1}$$

to solve for a₁

$$a_4 = a_1 \left(\frac{2}{3}\right)^{4-1}$$

$$\frac{5}{2} = a_1 \left(\frac{2}{3}\right)^3$$

$$\frac{27}{8} \cdot \frac{5}{2} = a_1 \left(\frac{8}{27}\right) \cdot \frac{27}{8}$$

$$\frac{135}{16} = a_1$$

$$a_1 = \frac{135}{16} \quad \times \frac{2}{3}$$

$$a_2 = \frac{45}{8} \quad \times \frac{2}{3}$$

$$a_3 = \frac{15}{4}$$

Another way to do it, use $S_n = \frac{a_1(1-r^n)}{1-r}$

5. Find the sum $\sum_{n=1}^4 4\left(\frac{1}{3}\right)^{n-1}$

One Way to do it

$$4\left(\frac{1}{3}\right)^0 + 4\left(\frac{1}{3}\right)^1 + 4\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)^3$$

$$4\left(\frac{1}{3}\right)^0 + 4\left(\frac{1}{3}\right)^1 + 4\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)^3$$

$$4 + 4\left(\frac{1}{3}\right) + 4\left(\frac{1}{9}\right) + 4\left(\frac{1}{27}\right)$$

$$4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} = \frac{160}{27} = 5\frac{25}{27} \approx 5.93$$

6 Find the sum of the infinite geometric series $\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \dots$

$$r = \frac{2}{5}$$

$$a_1 = \frac{2}{5}$$

$$S = \frac{a_1}{1-r}$$

$$S = \frac{\frac{2}{5}}{1-\frac{2}{5}} = \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{3}$$

7-8. Use mathematical induction to prove that $2 + 4 + 6 + \dots + 2n = n(n+1)$ is true for all natural numbers n.

Step 1 S_1 is true

$$2 = 1(1+1) \rightarrow 2 = 2 \checkmark$$

Step 2 $S_K \rightarrow S_{K+1}$

$$2 + 4 + 6 + \dots + 2K = K(K+1)$$

Step 3 $(K+1)$

$$2 + 4 + 6 + \dots + 2(K+1) = (K+1)(K+1+1)$$

$$2 + 4 + 6 + \dots + (2K+2) = (K+1)(K+2)$$

Step 4 Add the next term on both sides of step 2

$$2 + 4 + 6 + \dots + 2K + (2K+2) = K(K+1) + 2K+2$$

$$= K^2 + K + 2K + 2$$

$$= K^2 + 3K + 2$$

$$= (K+1)(K+2)$$

Step 5 Match Step 4 w/ Step 3

∴ S_n is true for all numbers

9. Find first three terms of $(2x-3y)^8$ and then simplify these terms.

$$(2x)^8 + 8(2x)^7(-3y) + 28(2x)^6(-3y)^2$$

$$256x^8 + 8(128)x^7(-3y) + 28(64)x^6(-3y)^2$$

$$256x^8 + (-3072)x^7y + 16128x^6y^2$$

$$a_1 = 256x^8$$

$$a_2 = -3072x^7y$$

$$a_3 = 16128x^6y^2$$

1	1	2	1	
1	3	3	1	
1	4	6	4	1
1	5	10	10	5
1	6	15	20	15
1	7	21	35	35
1	8	28		

10. Evaluate $\binom{120}{117}$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$\frac{120!}{117!(3!)!} = \frac{120 \cdot 119 \cdot 118 \cdot 117!}{117! \cdot 3 \cdot 2 \cdot 1} = \frac{1685040}{6} = 280840$$