$$a\,f(bx \pm c) \pm d$$

| SHIFTS | | | |
|---|---|---|---|
| *Vertical* | *Description* | *Coordinate:* $(x, y)$ | $f(x)$ *transformation* |
| $f(x)+d$ | Add $d$ to each y-coordinate of $f(x)$ | $(x, y+d)$ | Shifts the graph up $\uparrow$ $d$ units. |
| $f(x)-d$ | Subtract $d$ from each y-coordinate of $f(x)$ | $(x, y-d)$ | Shifts the graph down $\downarrow$ $d$ units. |
| | | | |
| *Horizontal* | *Description* | *Coordinate:* $(x, y)$ | $f(x)$ *transformation* |
| $f(x+c)$ | Subtract $c$ from each x-coordinate of $f(x)$ | $(x-c, y)$ | Shifts the graph left $\leftarrow$ $c$ units. |
| $f(x-c)$ | Add $c$ to each x-coordinate of $f(x)$ | $(x+c, y)$ | Shifts the graph right $\rightarrow$ $c$ units. |

| STRETCHES AND COMPRESSIONS | | | |
|---|---|---|---|
| *Vertical* | *Coordinate points* $(x, y)$ | *Coordinate:* $(x, y)$ | $f(x)$ *transformation* |
| $a\,f(x)$ $a>1$ | Multiply each y-coordinate of $f(x)$ by $a$. | $(x, ay)$ | Stretches the graph vertically by a factor of $a$. |
| $a\,f(x)$ $0<a<1$ | Multiply each y-coordinate of $f(x)$ by $a$ | $(x, ay)$ | Compresses the graph vertically by a factor of $a$. |
| | | | |
| *Horizontal* | *Coordinate points* $(x, y)$ | *Coordinate:* $(x, y)$ | $f(x)$ *transformation* |
| $f(bx)$ $b>1$ | Multiply each x-coordinate of $f(x)$ by $\frac{1}{b}$. | $\left(\frac{1}{b}x, y\right)$ | Compresses the graph horizontally by a factor of $\frac{1}{b}$. |
| $f(bx)$ $0<b<1$ | Multiply each x-coordinate of $f(x)$ by $\frac{1}{b}$. | $\left(\frac{1}{b}x, y\right)$ | Stretches the graph horizontally by a factor of $\frac{1}{b}$. |

| REFLECTIONS | | | |
|---|---|---|---|
| | *Coordinate points* $(x, y)$ | *Coordinate:* $(x, y)$ | $f(x)$ *transformation* |
| $-f(x)$ | Multiply each y-coordinate of $f(x)$ by $-1$. | $(x, -y)$ | Reflects the graph about the x-axis. |
| $f(-x)$ | Multiply each x-coordinate of $f(x)$ by $-1$. | $(-x, y)$ | Reflects the graph about the y-axis. |

| ABSOLUTE VALUE | | | |
|---|---|---|---|
| | *Coordinate points* $(x, y)$ | *Coordinate:* $(x, y)$ | $f(x)$ *transformation* |
| $\lvert f(x)\rvert$ | All points that have a negative y-coordinate switch to become a positive number. The x-coordinates are not affected. | $(x, \lvert y\rvert)$ | All points below x-axis are reflected about the x-axis. |
| $f(\lvert x\rvert)$ | All points with a negative x-coordinate swaps its y-coordinate value with that of its positive counterpart. | $(x^-, y^-) \rightarrow (x^-, y^+)$ | All points to the right of the y-axis get reflected about the y-axis overwriting all points to the left of the y-axis. |