

PC Final Exam Review Answers

① $x + 5y = 8$
 $\downarrow \quad \quad \downarrow$
 $r \cos \theta + 5r \sin \theta = 8$
 $r(\cos \theta + 5 \sin \theta) = 8$
 $r = \frac{8}{\cos \theta + 5 \sin \theta}$

② $z_1 z_2 =$
 $4(7) (\cos(15+25)^\circ + i \sin(15+25)^\circ)$
 $= 28 (\cos 40^\circ + i \sin 40^\circ)$

③ $A = P \left(1 + \frac{r}{n}\right)^{nt}$
 semi-annually $n=2$
 $A = 5,000 \left(1 + \frac{0.065}{2}\right)^{2(6)}$
 $\$ 7,339.23$

quarterly $n=4$
 $A = 5,000 \left(1 + \frac{0.065}{4}\right)^{4(6)}$
 $\$ 7,361.79$

④ a) $\log_4 \frac{1}{4} = \log_4 4^{-1} = -1$

b) $\ln e^{3/x} = 3/x$

⑤ a) $8^{1-x} = 4^{x+2}$
 $2^{3(1-x)} = 2^{2(x+2)}$
 $3 - 3x = 2x + 4$
 $-5x = 1$
 $x = -\frac{1}{5}$

b) $9^x = 3^{-1/3}$
 $3^{2x} = 3^{-1/3}$
 $2x = -1/3$
 $x = -1/6$

c) $\log_2 (x+2) - \log_2 (x-5) = 3$

$$\frac{(x+2)}{(x-5)} = 2^3$$

$$x+2 = 8(x-5)$$

$$x+2 = 8x-40$$

$$-7x = -42$$

$$x = 6$$

d) $\ln(x-4) + \ln(x+1) = \ln(x-8)$

$$(x-4)(x+1) = x-8$$

$$x^2 - 3x - 4 = x - 8$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0$$

$$x = 2$$

$$(6) \quad 70 = 100 \left(\frac{1}{2}\right)^{t/12}$$

$$0.70 = \left(\frac{1}{2}\right)^{t/12}$$

$$\ln 0.70 = \frac{t}{12} \ln 0.5$$

$$\frac{12 \ln 0.70}{\ln 0.5} = t$$

$$t = 6 \text{ hours}$$

$$(7) \quad \begin{cases} xy = 3 \\ x^2 + y^2 = 10 \end{cases}$$

$$y = \frac{3}{x}$$

$$\left(\frac{9}{x^2} + x^2\right) = (10)x^2$$

$$9 + x^4 = 10x^2$$

$$x^4 - 10x^2 + 9 = 0$$

$$(x^2 - 9)(x^2 - 1)$$

$$(x+3)(x-3)(x+1)(x-1)$$

$$-3 \quad 3 \quad -1 \quad 1$$

- $(-3, -1)$
- $(3, 1)$
- $(-1, -3)$
- $(1, 3)$

$$(7b) \quad \begin{cases} 3x^2 - 2y^2 = 1 \\ 4x - y = 3 \rightarrow y = 4x - 3 \end{cases}$$

$$3x^2 - 2(4x - 3)^2 = 1$$

$$3x^2 - 2(16x^2 - 24x + 9) = 1$$

$$3x^2 - 32x^2 + 48x - 18 = 1$$

$$-29x^2 + 48x - 19 = 0$$

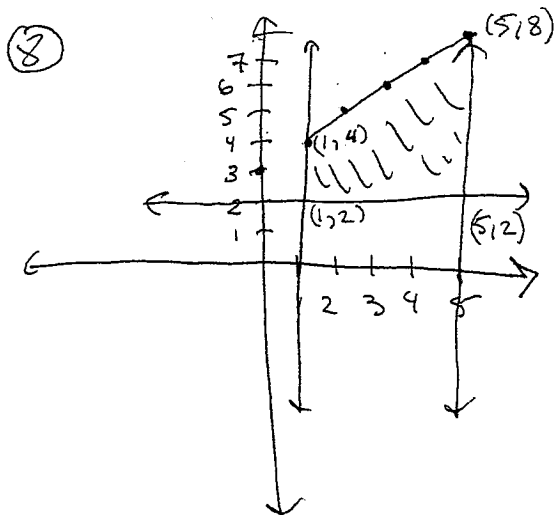
$$29x^2 - 48x + 19 = 0$$

$$(x - 1)(29x - 19)$$

$$1 \quad 19/29$$

$$(1, 1)$$

$$\left(\frac{19}{29}, \frac{11}{29}\right)$$



$$x - y \geq -3 \quad \text{Find edge points}$$

$$x + 3 \geq y \quad (1, 4) \text{ min}$$

$$y \leq x + 3 \quad (1, 2)$$

$$(5, 2) \text{ max}$$

$$(5, 8)$$

$$Z = 3x - 2y$$

$$Z = 3(1) - 2(4) = -5 \text{ min}$$

$$Z = 3(1) - 2(2) = -1$$

$$Z = 3(5) - 2(2) = 11 \text{ max}$$

$$Z = 3(5) - 2(8) = -1$$

⑨ regular $\rightarrow r$ deluxe $\rightarrow d$

$$\begin{cases} r \geq 50 \\ d \geq 75 \\ r+d \leq 150 \end{cases} \quad P = 200r + 250d$$

⑩ $\left[\begin{array}{ccc|c} 2 & 2 & 7 & -1 \\ 2 & 1 & 2 & 2 \\ 4 & 6 & 1 & 15 \end{array} \right] \xrightarrow[\begin{smallmatrix} R_2 - R_1 \\ R_2 \end{smallmatrix}]{}$ $\left[\begin{array}{ccc|c} 2 & 2 & 7 & -1 \\ 0 & -1 & -5 & 3 \\ 4 & 6 & 1 & 15 \end{array} \right]$

$\xrightarrow[\begin{smallmatrix} R_3 - 2R_2 \\ R_3 \end{smallmatrix}]{}$ $\left[\begin{array}{ccc|c} 2 & 2 & 7 & -1 \\ 0 & -1 & -5 & 3 \\ 0 & 4 & -3 & 11 \end{array} \right] \xrightarrow{\frac{1}{2}R_1}$ $\left[\begin{array}{ccc|c} 1 & 1 & 7/2 & -1/2 \\ 0 & -1 & -5 & 3 \\ 0 & 4 & -3 & 11 \end{array} \right]$

$\xrightarrow[\begin{smallmatrix} R_1 + R_2 \\ R_3 + 4R_2 \end{smallmatrix}]{}$ $\left[\begin{array}{ccc|c} 1 & 0 & -1.5 & 2.5 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & -23 & 23 \end{array} \right] \xrightarrow{R_3 / -23}$ $\left[\begin{array}{ccc|c} 1 & 0 & -1.5 & 2.5 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 1 & -1 \end{array} \right]$

$\xrightarrow[\begin{smallmatrix} R_2 - 5R_3 \\ R_1 + 1.5R_3 \end{smallmatrix}]{}$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right]$

Answer: (1, 2, -1)

⑪ $\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 2-4 & 1+6 & 4-2 \\ 4-8 & 2+12 & 8-4 \\ 6+2 & 3-3 & 12+1 \end{bmatrix} = \begin{bmatrix} -2 & 7 & 2 \\ -4 & 14 & 4 \\ 8 & 0 & 13 \end{bmatrix}$

⑫ $\det \begin{bmatrix} 7 & 9 \\ -2 & -5 \end{bmatrix} = 7(-5) - (-2)(9) = -35 + 18 = \boxed{17}$

⑬ see next page ⑭ yes draw picture

⑮ (a) ellipse (b) hyperbola

⑯ $\lim_{x \rightarrow 3} \frac{1}{x-2} = \frac{1}{3-2} = 1$

⑰ $\lim_{x \rightarrow -5} \frac{x^2 - 25}{x - 4} = \frac{0}{-9} = 0$

⑱ $\lim_{x \rightarrow -5} \frac{(x^2 - 25)}{x - 5} = \lim_{x \rightarrow -5} \frac{(x-5)(x+5)}{(x-5)} = -5 + 5 = 0$

(13)

$$\left[\begin{array}{ccc|cc} 1 & 3 & -4 & 0 & 1 & 0 \\ 2 & 4 & -4 & 1 & 0 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array} \right] \xrightarrow[\rightarrow R_3]{R_2 - R_3} \left[\begin{array}{ccc|cc} 1 & 3 & -4 & 0 & 1 & 0 \\ 2 & 4 & -4 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 \end{array} \right]$$

$$\xrightarrow{R_3 / -1} \left[\begin{array}{ccc|cc} 1 & 3 & -4 & 0 & 1 & 0 \\ 2 & 4 & -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right] \xrightarrow[R_1]{R_2 - R_1} \left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & -1 & 0 \\ 2 & 4 & -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_2 / 2} \left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & -1 & 0 \\ 1 & 2 & -2 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right] \xrightarrow[R_2]{R_1 - R_2} \left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & -1 & 0 \\ 0 & -1 & 2 & 0.5 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

second

$$\xrightarrow{R_2 / -1} \left[\begin{array}{ccc|cc} 1 & 0 & 2 & 1.5 & -2 & 0 \\ 0 & 1 & -2 & -0.5 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right] \quad A^{-1} = \begin{bmatrix} 3.5 & -2 & -2 \\ -2.5 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

first

$$\xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|cc} 1 & 0 & 2 & 1.5 & -2 & 0 \\ 0 & 1 & -2 & -0.5 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right] \quad \downarrow$$

$$\xrightarrow{R_1 - 2R_3} \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 3.5 & -2 & -2 \\ 0 & 1 & 0 & -2.5 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_2 + 2R_3} \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 3.5 & -2 & -2 \\ 0 & 1 & 0 & -2.5 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

(17)

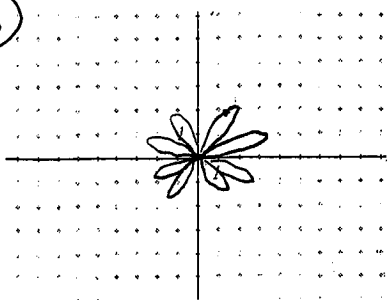
$$\lim_{h \rightarrow 0} \frac{(x+h)^2 - 4(x+h) + 7 - x^2 + 4x - 7}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{4x} - 4h + \cancel{7} - \cancel{x^2} + \cancel{4x} - \cancel{7}}{h}$$

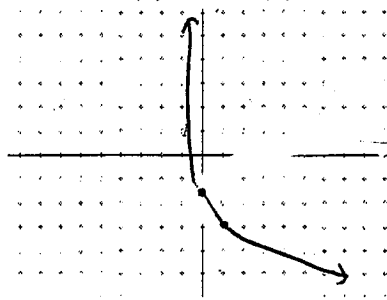
$$= \lim_{h \rightarrow 0} \frac{h(h + 2x - 4)}{h} = \lim_{h \rightarrow 0} h + 2x - 4 = 2x - 4$$

$$\text{So } f'(x) = 2x - 4 \text{ and } f'(2) = 0$$

18 Graph $r = 3\sin 4\theta$

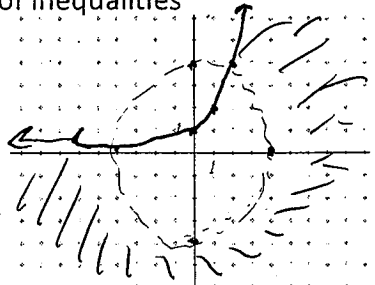


19 Graph $f(x) = -\log_2(x+1) - 2$

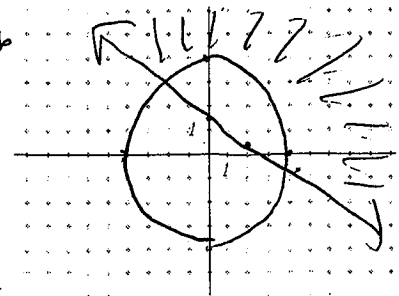


20 Graph the system of inequalities

(A) $x^2 + y^2 > 16$
 $y \leq 2^x$



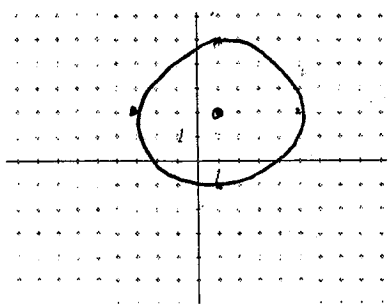
(B) $x^2 + y^2 \geq 16 \quad 0 \geq 16$
 $2x + 4y \geq 6 \quad 0 \geq 6$
 \downarrow
 $x + 2y \geq 3$
 $2y \geq -x + 3$
 $y \geq -\frac{x}{2} + \frac{3}{2}$



Graph each of the following conic sections. Identify all key characteristics such as center, vertex, focus(foci), directrix, major axis, minor axis, and equations of asymptotes where applicable.

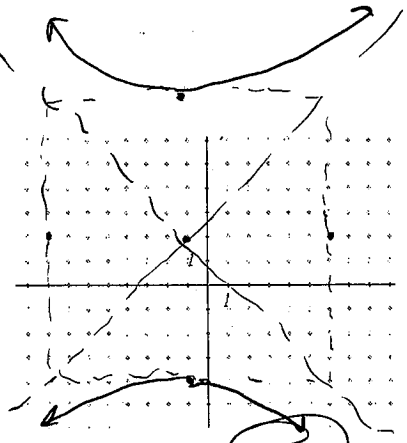
21 a. $\frac{(x-1)^2}{16} + \frac{(y-2)^2}{9} = 1$

Center $(1, 2)$
 major axis: 8
 minor axis: 6
 foci $(1 \pm \sqrt{7}, 2)$



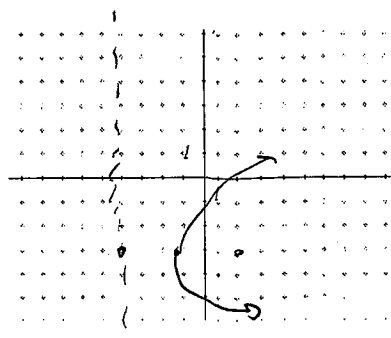
22 b. $\frac{(y-2)^2}{36} - \frac{(x+1)^2}{49} = 1$

Center $(-1, 2)$
 foci $(-1, 2 \pm \sqrt{85})$



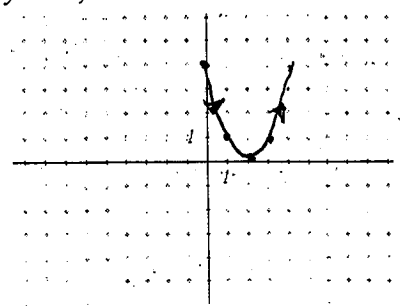
23 c. $(y+3)^2 = 12(x+1)$

$4f = 12$
 $f = 3$
 Center $(-1, -3)$
 directrix $x = -4$
 focus $(2, -3)$



24 Graph the plane curve described by the parametric equations $x = t + 2$, $y = t^2$, for $-2 \leq t \leq 2$.

t	x	y
-2	0	4
-1	1	1
0	2	0
1	3	1
2	4	4



25 Graph $f(x) = \frac{1}{x+2}$

Find the following limits:

- a. $\lim_{x \rightarrow 2^-} f(x) = -\infty$
- b. $\lim_{x \rightarrow 2^+} f(x) = +\infty$
- c. $\lim_{x \rightarrow 2} f(x)$ DNE

