Pre-Calculus

Review 10:2-5

Name Key

Date

Hour

Use the following formulas when appropriate

$a_n = a_1 + (n-1)d$ $\quad S_n = \frac{n}{2}(a_1 + a_n)$ $\quad S_n = \frac{a_1(1-r^n)}{1-r}$ $\quad \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$\boxed{a_n = a_1 r^{n-1}}$ $\quad S_n = \frac{n}{2}[2a_1 + (n-1)d]$ $\quad S = \frac{a_1}{1-r}$

1: Given: 1, 5, 9, 13,....
Determine the common difference, the nth term, and the 100th term of the arithmetic sequence.

$\boxed{d=4}$

$a_n = 1 + (n-1)4$

$a_n = 1 + 4n - 4$

$\boxed{a_n = 4n - 3}$

$a_{100} = 4(100) - 3$

$\boxed{a_{100} = 397}$

2: The 12th term of an arithmetic sequence is 32, and the 5th term is 18. Find the common difference, the nth term, and the 20th term.

$\boxed{d=2}$

$a_1 = 10$

7 terms

5th term: 18

12th term: 32

14

$d = \frac{14}{7} = 2$

$a_{12} = a_1 + (12-1)2$

$32 = a_1 + (11)2$

$32 = a_1 + 22$

$-22 \quad -22$

$10 = a_1$

$a_n = 10 + (n-1)2$

$a_n = 10 + 2n - 2$

$\boxed{a_n = 2n + 8}$

$a_{20} = 2(20) + 8$

$\boxed{a_{20} = 48}$

3: Given: 144, 12, 1, $\frac{1}{12}$,......
Determine the common ratio, the fifth term, and the nth term of the geometry sequence.

$\frac{a_2}{a_1}$ $\frac{12}{144} = \boxed{\frac{1}{12} = r}$

$\boxed{a_n = 144\left(\frac{1}{12}\right)^{n-1}}$

$\boxed{a_5 = \frac{1}{144}}$

4: The common ratio in a geometric sequence is $\frac{2}{3}$, and the fourth term is $\frac{5}{2}$. Find the first three terms.

$a_4 = \frac{5}{2}$

$r = \frac{2}{3}$

$a_4 = a_1\left(\frac{2}{3}\right)^{4-1}$

$\frac{5}{2} = a_1\left(\frac{2}{3}\right)^3$

$\frac{5}{2} = a_1\left(\frac{8}{27}\right)$

$\frac{27}{8} \cdot \frac{5}{2} = a_1\left(\frac{8}{27}\right) \cdot \frac{27}{8}$

$a_1 = \frac{135}{16}$

$a_1 = \frac{135}{16}$

$a_2 = \frac{45}{8}$

$a_3 = 15/4$

5. Find the sum $\sum_{n=1}^{4} 4\left(\frac{1}{3}\right)^{n-1}$

$4\left(\frac{1}{3}\right)^{1-1}$ $4\left(\frac{1}{3}\right)^{2-1}$ $4\left(\frac{1}{3}\right)^{3-1}$

$4\left(\frac{1}{3}\right)^{1}$ $4\left(\frac{1}{3}\right)^{2}$

$r = \frac{1}{3}$ $a_1 = 4$

$a_2 = \frac{4}{3}$ $a_3 = \frac{4}{9}$ $a_4 = \frac{4}{27}$

$S_n = \frac{a_1(1-r^n)}{1-r} =$

$S_4 = \frac{4\left(1-\left(\frac{1}{3}\right)^4\right)}{1-1/3} = \frac{4\left(1-\frac{1}{81}\right)}{2/3} = \frac{4\left(\frac{80}{81}\right)}{2/3} = \boxed{160/27}$

6 Find the sum of the infinite geometric series $\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \ldots\ldots$.

$\frac{\frac{4}{25}}{\frac{2}{5}} = \frac{4}{25} \cdot \frac{5}{2} = \boxed{\frac{2}{5}} = r$

$S = \frac{a_1}{1-r} = \frac{2/5}{1-2/5} = \frac{2/5}{3/5} = \frac{2}{\cancel{5}} \cdot \frac{\cancel{5}}{3} = \boxed{\frac{2}{3}}$

7-8. Use mathematical induction to prove that 2 + 4 + 6 + … + 2n = n (n + 1) is true for all natural numbers n.

S_n

$\boxed{1}$ $2 = 1(1+1) \rightarrow 2 = 1(2) \rightarrow 2 = 2$ ✓

$\boxed{2}$ $S_K = 2+4+6+\ldots+2K = K(K+1)$

$\boxed{3}$ $S_{K+1} = 2+4+6+\ldots \underbrace{\boxed{+2(K+1)}}_{\text{Next term}} = \boxed{(K+1)(K+2)}$

④ $2+4+6+\ldots+2K + 2(K+1) = K(K+1) + 2(K+1)$

$= K^2 + K + 2K + 2$

$= K^2 + 3K + 2$

$= (K+1)(K+2)$

we have a match

∴ S_n is true for all positive integers.

9. Find first three terms of $(2x-3y)^8$ and then simplify these terms.

$1(2x)^8(-3y)^0$ $8(2x)^7(-3y)^1$ $\underline{28}(2x)^6(-3y)^2$

$256x^8$ $8(128)x^7(-3)y$ $28(64)x^6(9)y^2$

$-3072x^7y$ $16128x^6y^2$

1
1 1
1 2 1
1 3 3
1 4 6
1 5 10
1 6 15
1 7 21
1 8 28

10. Evaluate $\binom{120}{117} = \frac{120!}{117!(120-117)!}$

$\frac{120!}{117!\,3!} = \frac{120 \cdot 119 \cdot 118 \cdot \cancel{117!}}{\cancel{117!}\ 3 \cdot 2 \cdot 1} = \frac{1685040}{6} = \boxed{280840}$

For #9, instead of using Pascal's triangle, you can use $\binom{n}{r} a^{n-r} b^r$