

Warm-Up

$$\text{Multiply, if possible. } A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 2 & 0 \\ -1 & -3 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$A \cdot B$
 $3 \times 2 \quad 2 \times 3$

1) AB

$B \cdot A$
 $2 \times 3 \quad 3 \times 2$

2) BA

$B \cdot C$
 $2 \times 3 \quad 3 \times 1$

3) BC

$$\begin{bmatrix} 2 & -8 & 20 \\ 8 & 3 & 5 \\ 10 & 2 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 14 \\ 9 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 21 \end{bmatrix}$$

Identity Matrix

The identity matrix is a "square" matrix with 1's down the main diagonal and 0's every else.

Multiplicative Inverse

When you multiply a matrix to its inverse matrix, you will get the identity matrix.

$$A \cdot A^{-1} = I \quad \text{and} \quad A^{-1} \cdot A = I$$

Ex1) Are matrix A and matrix B inverses of each other?

$$A = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \times 2 \\ [-1 \ 3] \\ [2 \ -5] \end{bmatrix} \cdot \begin{bmatrix} 2 \times 2 \\ [5 \ 3] \\ [2 \ 1] \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{yes } A \& B \text{ are inverses.}$$

Shortcut for finding the multiplicative inverse of a 2×2 matrix.

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then } A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Ex2) Find the multiplicative inverse of

$$A = \begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix} \quad a = -1 \quad b = -2 \quad c = 3 \quad d = 4$$

$$A^{-1} = \frac{1}{(-1)(4) - (-2)(3)} \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$$

$$A^{-1} = \boxed{\begin{bmatrix} 2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}}$$

To find the multiplicative inverse of any other "Square" matrix, create an augmented matrix with both the original matrix and the identity matrix, then perform Gauss-Jordan elimination.

Ex3) Find the multiplicative inverse of $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & +3 & 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & 1 & 0 \\ -2 & +3 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{2R_1+R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & 1 & 0 \\ 0 & -5 & 2 & 2 & 0 & 1 \end{array} \right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & -5 & 2 & 2 & 0 & 1 \end{array} \right] \xrightarrow{5R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{5}{2} & 0 & -\frac{5}{2} & 1 \end{array} \right]$$

$$\xrightarrow{-2R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -4 & 5 & -2 \end{array} \right] \xrightarrow{\frac{1}{2}R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 2 & -1 \\ 0 & 0 & 1 & -4 & 5 & -2 \end{array} \right] \xrightarrow{-R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 5 & -5 & 2 \\ 0 & 1 & 0 & -2 & 2 & -1 \\ 0 & 0 & 1 & -4 & 5 & -2 \end{array} \right] \xrightarrow{R_2+R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & 1 \\ 0 & 1 & 0 & -2 & 2 & -1 \\ 0 & 0 & 1 & -4 & 5 & -2 \end{array} \right]$$

$$A^{-1} = \left[\begin{array}{ccc} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{array} \right]$$

Lets check our answer in the calculator.

Find the inverse of $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{bmatrix}$

This matrix has an order of 3x6.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & 1 & 0 \\ -2 & -3 & 0 & 0 & 0 & 1 \end{array} \right]$$

TI-Nspire Directions

- 1) New doc - don't save - Calculator
- 2) Menu - Matrix - Reduced Row Echelon Form
- 3) Menu - Matrix - Create
- 4) Create your matrix and hit enter

TI-84

- 1) 2nd, (x^{-1}) Matrix, move right to edit, enter
- 2) Create Matrix A, 2nd, (Mode) Quit
- 3) 2nd, (x^{-1}) Matrix, move right to MATH, down to rref(), enter
- 4) 2nd, (x^{-1}) Matrix, enter
- 5) Enter