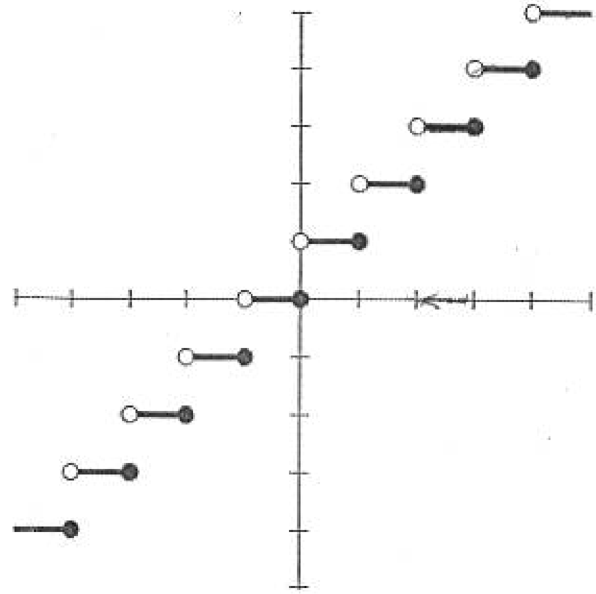
Pre-Calculus

Test 11 Review

Name Key

Date   Hour_____

1. Refer to the graph at the right and determine the following values or limits, if they exist. If the limit does not exist explain why.



A) $\lim\limits_{x \to 2^+} f(x)$ 3

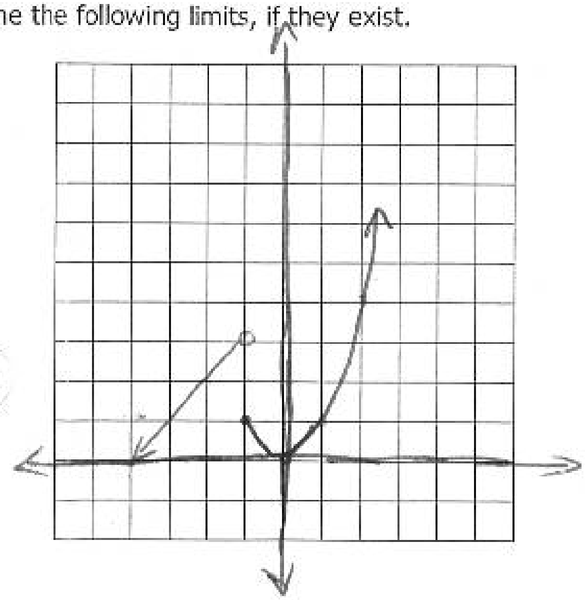
B) $\lim\limits_{x \to 2^-} f(x)$ 2

C) $\lim\limits_{x \to 2} f(x)$ DNE

D) f(1) 1

2. Graph the function $f(x)$ at the right to determine the following limits, if they exist. If the limits do not exist explain why.

$$f(x) = \begin{cases} x^2 & \text{if } x \ge -1 \\ x+4 & \text{if } x < -1 \end{cases}$$



A) $\lim\limits_{x \to -1^+} f(x)$ 1

B) $\lim\limits_{x \to -1^-} f(x)$ 3

C) $\lim\limits_{x \to -1} f(x)$ DNE

D) $\lim\limits_{x \to 3} f(x)$ 9

3-8. Evaluate the limits (if they exist)

3. $\lim_{x \to 5} \frac{x+1}{x-3}$ $\quad \frac{5+1}{5-3} = \frac{6}{2}$ $\quad 3$

4. $\lim_{x \to -2} (x^3 - 3x + 6)$ $\quad (-2)^3 - 3(-2) + 6$ $\quad -8 + 6 + 6$ $\quad -8 + 12$ $\quad 4$

5. $\lim_{x \to 3} \frac{x^2 + x - 12}{x - 3}$ $\quad \frac{\cancel{(x-3)}(x+4)}{\cancel{x-3}}$ $\quad 7$

6. $\lim_{u \to 0} \frac{(u+1)^2 - 1}{u}$ $\quad \frac{u^2 + 2u + \cancel{1} - \cancel{1}}{u}$ $\quad \cancel{\frac{u(u+2)}{u}}$ $\quad 2$

7. $\lim_{x \to 1} \frac{4x^2 - 3x + 5}{6 + 5x - 3x^2}$ $\quad \frac{4(1)^2 - 3(1) + 5}{6 + 5(1) - 3(1)^2}$ $\quad \frac{4 - 3 + 5}{6 + 5 - 3} = \frac{6}{8} = \boxed{\frac{3}{4}}$

8. $\lim_{x \to 9} \frac{\sqrt{x+7} - 3}{5}$ $\quad \frac{\sqrt{9+7} - 3}{5} = \frac{\sqrt{16} - 3}{5}$ $\quad \frac{4-3}{5} = \boxed{\frac{1}{5}}$

9. Find the slope of the tangent to the graph of $f(x) = x^2 - 4x$ at the point $(\overset{a}{3}, \overset{f(a)}{-3})$, then write the equation of the tangent line at that point.

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{(a+h)^2 - 4(a+h) - \overset{f(a)}{-3}}{h} = \lim_{h \to 0} \frac{(3+h)^2 - 4(3+h) + 3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{9} + 6h + h^2 \cancel{-12} - 4h \cancel{+3}}{h} = \lim_{h \to 0} \frac{h^2 + 2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(h+2)}{\cancel{h}} = 2 = m_{tan}$$

$y + 3 = 2(x - 3)$

$y + 3 = 2x - 6$

$\boxed{y = 2x - 9}$

10. A) Find the derivative of $f(x) = 2x^2 - 1$ B) Determine the slope of the tangent line at 3

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^2 - 1 - (2x^2 - 1)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 1 - 2x^2 + 1}{h} = \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + h^2 \cancel{-2x^2} \cancel{+1}}{h}$$

$$= \lim_{h \to 0} \frac{4xh + h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(4x + h)}{\cancel{h}} = 4x \quad \boxed{f'(x) = 4x}$$

$f'(3) = 4(3)$

$\boxed{m_{tan} \text{ @ } 3 \text{ is } 12}$

11. Determine for what numbers, if any, the given function is discontinuous.

$(x) = \frac{x+1}{(x+1)(x-4)}$

① $f(a) = \overset{real}{\#}$

② $\lim_{x \to a} f(x)$

③ step 1 = step 2

try $x = -1$

$f(-1) = \frac{-1+1}{(-1+1)(-1-4)} = \frac{0}{0(-5)} = \frac{0}{0}$

UND

$x = 4$

$f(4) = \frac{4+1}{(4+1)(4-4)} = \frac{5}{5(0)}$

$\frac{5}{0}$ = UND

Discontinuous at -1 and 4 because they are both undefined.