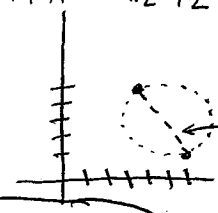
Conic Sections - Circles and Ellipses Name: Key

1. Find the equation of a circle with diameter endpoints (4, 5) and (6, 1).

x_1, y_1 x_2, y_2

Circle
Diam: $2\sqrt{2}$
radius: $\sqrt{2}$
Center: $(5,3)$

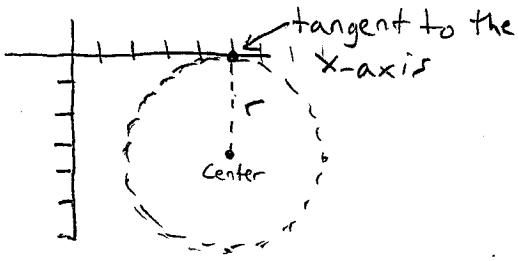
$d=\sqrt{(6-4)^2+(1-5)^2}$
$=\sqrt{2^2+(-4)^2}$
$=\sqrt{4+16}$
$=\sqrt{20}$
$=2\sqrt{2}$



Center
$\left(\frac{4+6}{2}, \frac{5+1}{2}\right)$
$\left(\frac{10}{2}, \frac{6}{2}\right)$
$(5,3)$

$(x-5)^2+(y-3)^2=2$

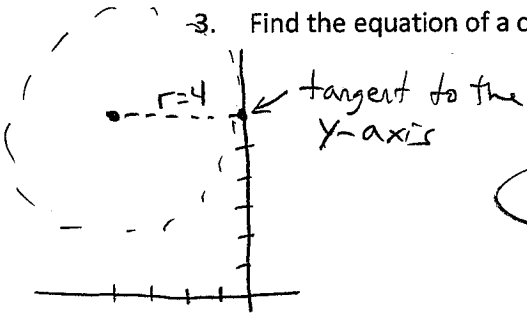
2. Find the equation of a circle with center (5, -3) and tangent to the x-axis.



radius: 3

$(x-5)^2+(y+3)^2=9$

3. Find the equation of a circle with center (-4, 6) and tangent to the y-axis.



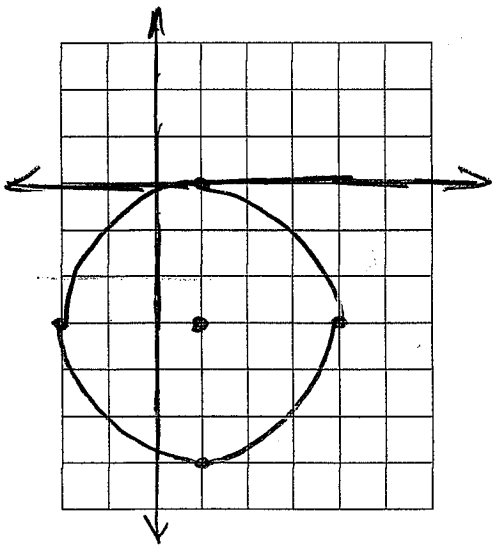
radius = 4

$(x+4)^2+(y-6)^2=16$

4. Find the center, radius of the circle, and graph. the circle.

$(x-1)^2+(y+3)^2=9$

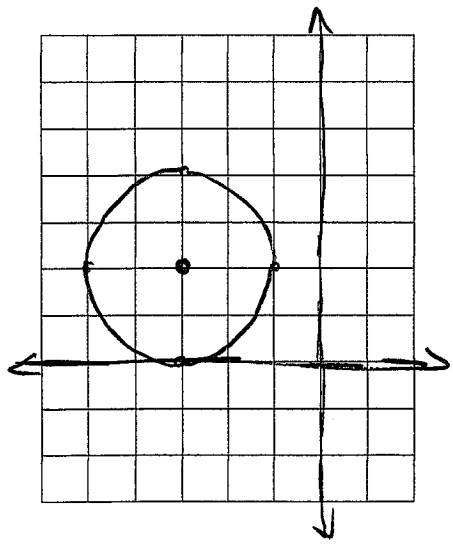
Center: $(1,-3)$
radius: 3



6. Find the center, radius, and graph of

$(x+3)^2+(y-2)^2=4$

Center: $(-3,2)$
radius: 2



5. Find the standard form of the equation of the ellipse with a horizontal major axis of length 8 and minor axis of length 4, center at (-1, 3)

$a=4$

$b=2$

$$\frac{(x+1)^2}{4}+\frac{(y-3)^2}{2}=1$$

6. Find the center, vertices, co-vertices, then find the

foci, and graph of the ellipse.

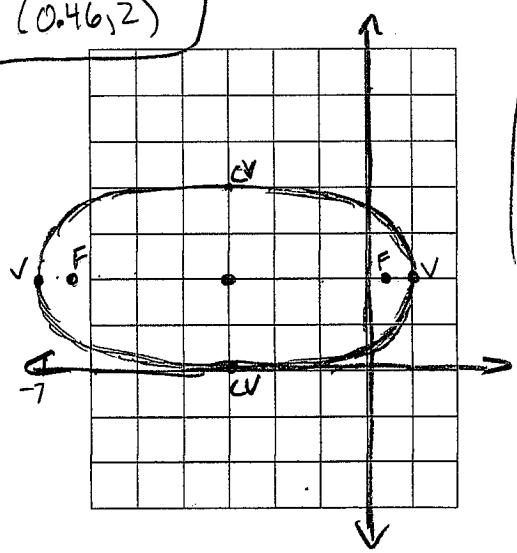
$$\frac{2(x+3)^2}{32}+\frac{8(y-2)^2}{32}=\frac{32}{32}$$

$$\frac{(x+3)^2}{16}+\frac{(y-2)^2}{4}=1$$

Center: $(-3,2)$
Vertices: $(1,2)$
$(-7,2)$
Co-Vertices: $(-3,4)$
$(-3,0)$
Foci: $(0.46,2)$

$a=4$
$b=2$
$c=2\sqrt{3}$
$c\approx 3.46$

$c^2=a^2-b^2$
$c^2=16-4$
$\sqrt{c^2}=\sqrt{12}$
$c=2\sqrt{3}$



Foci: $-3+3.46=0.46$

$-3-3.46=-6.46$

8. Find the standard form of the ellipse,

center, vertices, co-vertices, foci, and graph.

$$9x^2+25y^2-36x-50y-164=0$$

$9x^2-36x \quad +25y^2-50y \quad =164$

Complete the square

$9(x^2-4x \quad)+25(y^2-2y \quad)=164$

$\left(\frac{-4}{2}\right)^2=(-2)^2=4 \quad \times 9 \qquad \left(\frac{-2}{2}\right)^2=(-1)^2=1 \quad \times 25$

$+25$
$+36$

$9(x^2-4x+4)+25(y^2-2y+1)=164+25+36$

$$\frac{9(x-2)^2}{225}+\frac{25(y-1)^2}{225}=\frac{225}{225}$$

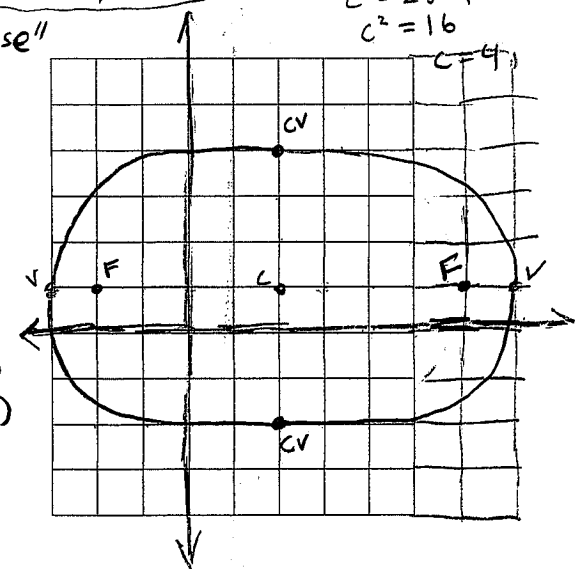
standard form $\rightarrow$ $\boxed{\frac{(x-2)^2}{25}+\frac{(y-1)^2}{9}=1}$

To Find "c"
$c^2=a^2-b^2$
$c^2=25-9$
$c^2=16$
$c=4$

"Horizontal Ellipse"

$a=5$
$b=3$
$c=4$

Center: $(2,1)$
Vertices: $(7,1)$
$(-3,1)$
CoVertices: $(2,4)$
$(2,-2)$
Foci: $(6,1)$
$(-2,1)$



9. Write the equation in standard form.

[redacted].

$$4x^2+25y^2-24x+100y+36=0$$

$4x^2-24x+25y^2+100y=-36$

Completing the square

$4(x^2-6x\quad)+25(y^2+4y\quad)=-36$

$4(x^2-6x+9)+25(y^2+4y+4)=-36+36+100$

$\frac{4(x-3)^2}{100}+\frac{25(y+2)^2}{100}=\frac{100}{100}$

$\boxed{\frac{(x-3)^2}{25}+\frac{(y+2)^2}{4}=1}$

$\left(\frac{-6}{2}\right)^2$
$(-3)^2$
9

$\left(\frac{4}{2}\right)^2$
$(2)^2$
4

10. Find the solution of the system of equations by graphing

$x^2+y^2=1$ → Circle center $(0,0)$ radius $=1$

$\frac{x^2}{9}+\frac{9y^2}{9}=\frac{9}{9}$

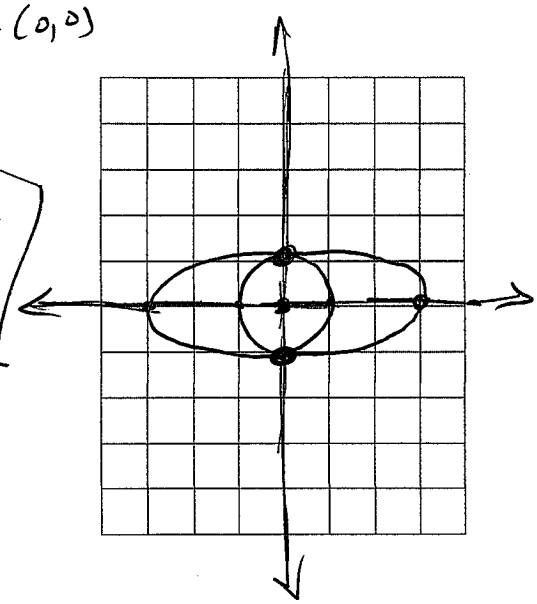
$\frac{x^2}{9}+\frac{y^2}{1}=1$

"Horiz. Ellipse"

$a=3$

$b=1$

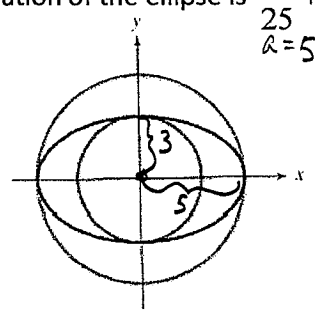
center: $(0,0)$



The solutions of the system are $(0,1)$ & $(0,-1)$

The equation of the ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$. Write an equation for

$a=5 \quad b=3$



Small circle → radius of 3
center $(0,0)$
Equation $x^2+y^2=9$

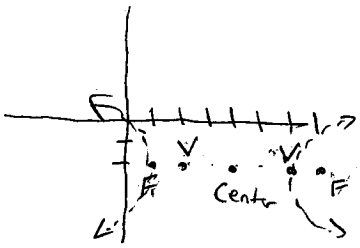
Large Circle → radius of 5
Center $(0,0)$.
Equation $x^2+y^2=25$

Conic Sections - Hyperbolas

Name: Key

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

1. Find the equation of a hyperbola with Center (4, -2), Focus (7, -2) and Vertex (6, -2).



$a = 2$

$b = \sqrt{5}$

$c = 3$

$c^2 = a^2 + b^2$

$9 = 4 + b^2$

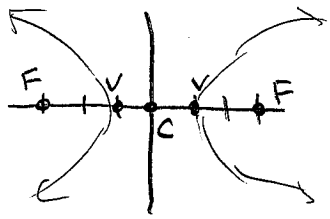
$-4 \quad -4$

$\sqrt{5} = \sqrt{b^2}$

$b = \sqrt{5}$

$$\frac{(x-4)^2}{4} - \frac{(y+2)^2}{5} = 1$$

2. Find the equation of a hyperbola with Foci (0,-3), (0,3) and Vertices (0,-1), (0, 1).



$a = 1$

$b = 2\sqrt{2}$

$c = 3$

$c^2 = a^2 + b^2$

$9 = 1 + b^2$

$-1 \quad -1$

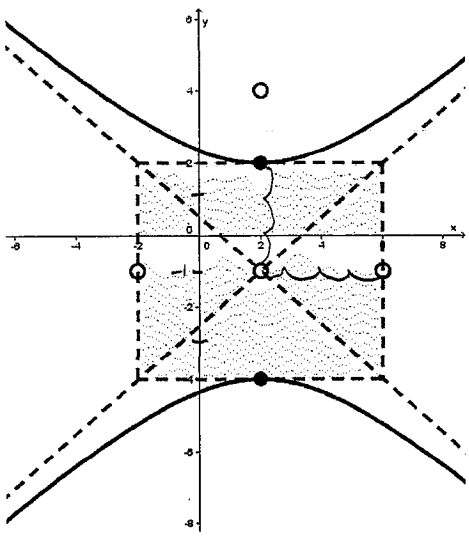
$\sqrt{8} = \sqrt{b^2}$

$b = 2\sqrt{2}$

Center: $(0,0)$

$$\frac{x^2}{1} - \frac{y^2}{8} = 1$$

3. Find the equation of the hyperbola below:



"Vertical"

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$a = 3$

$b = 4$

Center: $(2,-1)$

$$\frac{(y+1)^2}{9} - \frac{(x-2)^2}{16} = 1$$

"Horizontal"

4. Find the center, vertices, asymptotes, foci and graph.

$$\frac{(x+4)^2}{9}-\frac{(y+3)^2}{16}=1$$

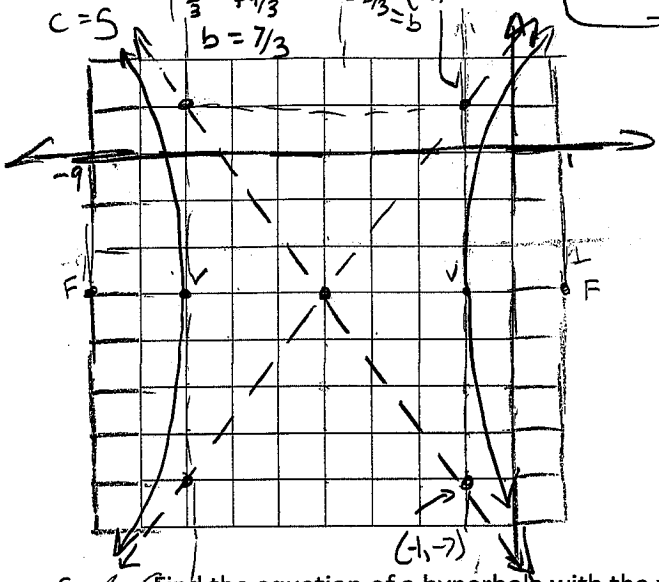
$a=3$
$b=4$
$c=5$

$c^2=9+16$
$c^2=25$
$c=5$

Asym
slope: $4/3$
point: $(-1,1)$
$y=4/3x+b$
$1=4/3(-1)+b$
$1=-4/3+b$
$\frac{3}{3}+4/3$
$b=7/3$

Asym
slope: $-\frac{4}{3}$
point $(-1,-7)$
$-7=-\frac{4}{3}(-1)+b$
$-\frac{21}{3}=\frac{-4}{3}+b$
$-4/3 \quad -4/3$
$-25/3=b$

Center: $(-4,-3)$
Vertices: $(-7,-3)$
$(-1,-3)$
Foci: $(-9,-3)$
$(1,-3)$
Asym: $y=\frac{4}{3}x+\frac{7}{3}$
$y=-\frac{4}{3}x-\frac{25}{3}$



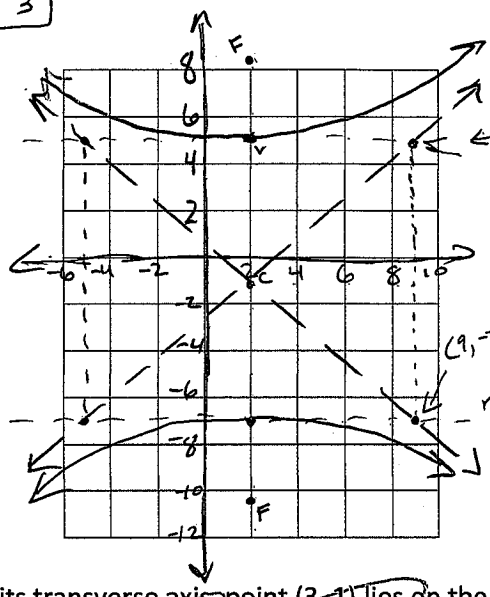
"Vertical"

5. Find the center, asymptotes, vertices, foci, and graph

$$\frac{(y-2)^2}{36}-\frac{(x+1)^2}{49}=1$$

$a=6$
$b=7$
$c=\sqrt{85}\approx 9.2$
$c^2=36+49$
$\sqrt{c^2}=\sqrt{85}$

Center: $(2,-1)$
Vertices: $(2,5)$
$(2,-7)$
Foci: $(2,8.2)$
$(2,-10.2)$
Asym: $y=\frac{6}{7}x-\frac{19}{7}$
$y=-\frac{6}{7}x+\frac{5}{7}$



Asym
$m=\frac{6}{7}$ $(9,5)$
$5=\frac{6}{7}(9)+b$
$\frac{35}{7}=\frac{54}{7}+b$
$-\frac{54}{7} \quad -\frac{54}{7}$
$b=-\frac{19}{7}$
$y=6/7x-\frac{19}{7}$

$m=-\frac{6}{7}$ $(9,-7)$
$-7=-\frac{6}{7}(9)+b$
$-\frac{49}{7}=-\frac{54}{7}+b$
$+\frac{54}{7} \quad +\frac{54}{7}$
$b=5/7$

6. Find the equation of a hyperbola with the x-axis as its transverse axis, point (3, 1) lies on the graph of this hyperbola and point (4, 2) lies on the asympotote of the hyperbola.

Skip #6

Convert the equation to standard form, then graph and find all necessary information. * Graphs on the next sheet

Center, vertices, foci & Asymptotes

7. $9y^2-4x^2-18y+24x-63=0$

$9y^2-18y-4x^2+24x=63$
$9(y^2-2y+1)-4(x^2-6x+9)=63+9-36$
$\frac{9(y-1)^2}{36}-\frac{4(x-3)^2}{36}=\frac{36}{36}$
$\frac{(y-1)^2}{4}-\frac{(x-3)^2}{9}=1$

$a=2$ $c^2=4+9$
$b=3$ $c^2=13$
$c=\sqrt{13}\approx 3.6$

* "Vertical"
w/ center: $(3,1)$
Vertices: $(3,3)$
$(3,-1)$
Foci: $(3,4.6)$
$(3,-2.6)$
Asym: $y=2/3x-1$
$y=2/3+3$

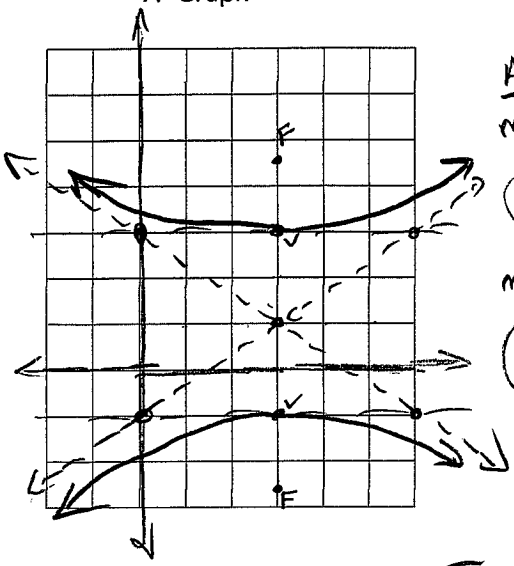
8. $4x^2-y^2+32x+6y+39=0$

$4x^2+32x-y^2+6y=-39$
$4(x^2+8x+16)-(y^2-6y+9)=-39+64-9$
$\frac{4(x+4)^2}{16}-\frac{(y-3)^2}{16}=\frac{16}{16}$
$\frac{(x+4)^2}{4}-\frac{(y-3)^2}{16}=1$

$a=2$
$b=4$
$c=\sqrt{20}=2\sqrt{5}\approx 4.5$
$c^2=4+16$
$c=\sqrt{20}=2\sqrt{5}$

"Horizontal"
Center: $(-4,3)$
Vertices: $(-6,3)$
$(-2,3)$
Foci: $(0.5,3)$
$(-8.5,3)$
Asym: $y=2x+11$
$y=-2x-5$

7. Graph



Asym

$m = \frac{2}{3}$ $b = -1$

$y = \frac{2}{3}x - 1$

$m = -\frac{2}{3}$ $b = 3$

$y = -\frac{2}{3}x + 3$

Asym

$m = \frac{4}{2} = \frac{2}{1}$

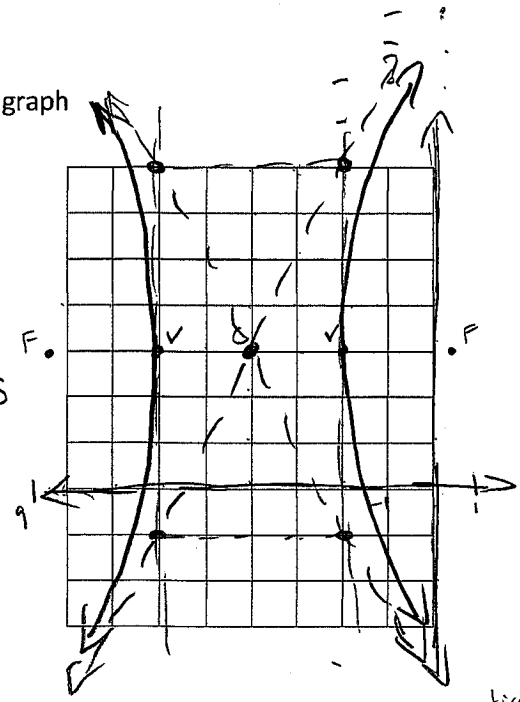
$b = 11$

$y = 2x + 11$

$m = -2$ $b = -5$

$y = -2x - 5$

8. graph



Find the solution to the system by graphing.

9.

Hyp → $\begin{cases} x^2 - y^2 = 4 \\ x^2 + y^2 = 4 \end{cases}$

Circle → $x^2 + y^2 = 4 \rightarrow r = 2$

$\frac{x^2}{4} - \frac{y^2}{4} = 1$ → "Horizontal"

center: $(0,0)$

$a = 2$ $b = 2$

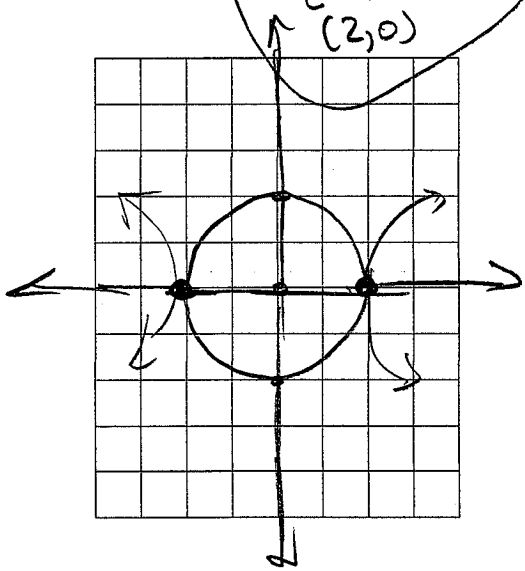
Vertices: $(-2,0)$

$(2,0)$

Solution

$(-2,0)$

$(2,0)$



10.

$\begin{cases} 9x^2 + y^2 = 9 \\ y^2 - 9x^2 = 9 \end{cases}$

$9x^2 + y^2 = 9 \rightarrow \frac{x^2}{1} + \frac{y^2}{9} = 1$ ← vertical ellipse

$y^2 - 9x^2 = 9 \rightarrow \frac{y^2}{9} - \frac{x^2}{1} = 1$ ← Vertical hyp

Center $(0,0)$

$a = 3$

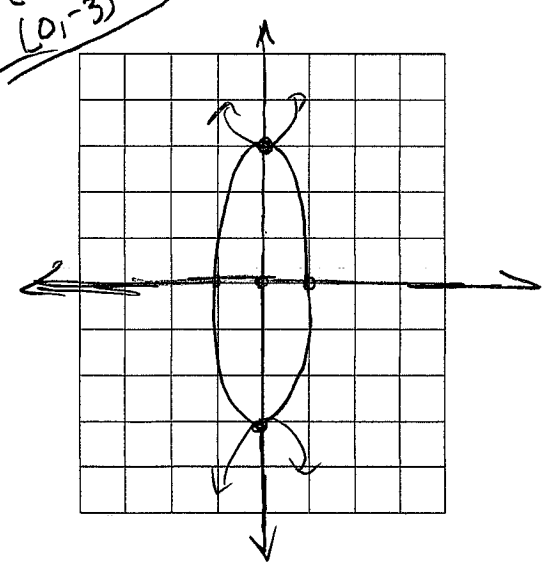
Vert: $(0,3)$

$(0,-3)$

Solution

$(0,3)$

$(0,-3)$



11. An architect designs two houses that are shaped and positioned like a part of the branches of the hyperbola whose equation is $625y^2 - 400x^2 = 250{,}000$, where x and y are in yards. How far apart are the houses at their closest point?

what is the length of the transverse axis? From vertex to vertex

$\frac{625y^2}{250{,}000} - \frac{400x^2}{250{,}000} = 1$

$\frac{y^2}{400} - \frac{x^2}{625} = 1$

$a = 20$

Center $(0,0)$

Vertices $(0,20)$

$(0,-20)$

The houses are 40 yards apart.

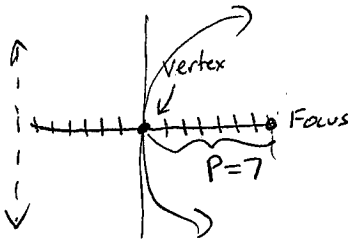
Conic Sections - Parabolas

"Horiz" $y^2=4px$

"Vert" $x^2=4py$

Name: Key

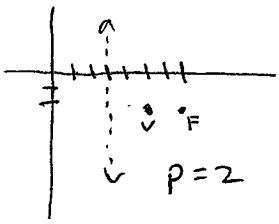
1. Find the equation of a parabola with focus (7, 0) and directrix x = -7.



$y^2=4(7)x$

$y^2=28x$

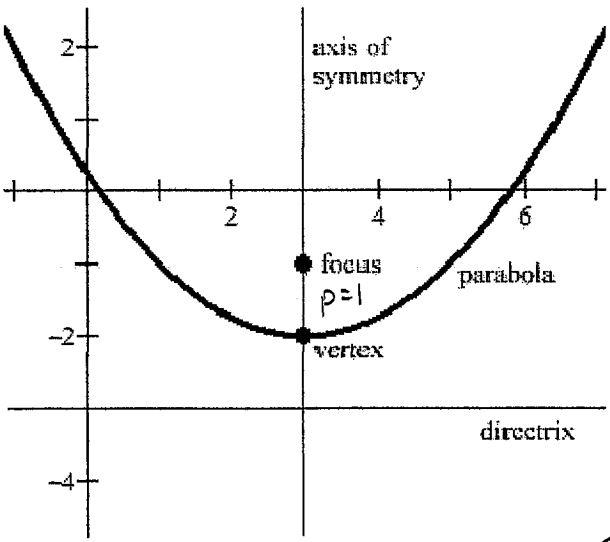
2. Find the equation of a parabola with focus (7, -2) and Vertex (5, -2).



$y^2=4(2)x$

$y^2=8x$

3. Find the equation of the parabola below:



Center: (3, -2)

p = 1

"Vertical"

$x^2=4py$

$(x-3)^2=4(1)(y+2)$

$(x-3)^2=4(y+2)$

"Vertical"

4. Find the Vertex, focus, directrix, and graph.

$(x-2)^2 = \underset{4p}{8}(y-1)$

$p = 2$

Vertex: $(2,1)$

Focus: $(2,3)$

Directrix: $y=-1$

plug $y=3$ in to find x_1 & x_2

from focus

$(x-2)^2 = 8(3-1)$

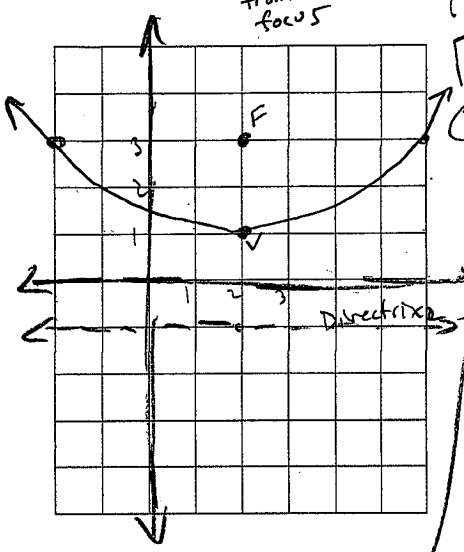
$\sqrt{(x-2)^2} = \sqrt{16}$

$(x-2) = \pm 4$

$+2 \quad +2$

$x = 6 \rightarrow (6,3)$

$x = -2 \rightarrow (-2,3)$



"Horizontal"

5. Find the vertex, focus, directrix, and graph.

$(y-1)^2 = \underset{4p}{-8}x$

$p = -2$

Vertex: $(0,1)$

Focus: $(-2,1)$

Directrix: $x=2$

plug $x=-2$ in solve for y's

$(y-1)^2 = -8(-2)$

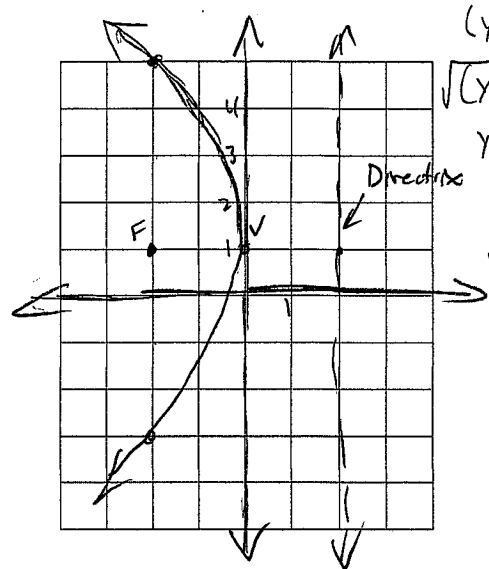
$\sqrt{(y-1)^2} = \sqrt{16}$

$y-1 = \pm 4$

$+1 \quad +1$

$y = 5 \quad (-2,5)$

$y = -3 \quad (-2,-3)$



Convert the equation to standard form, then graph and find all necessary information.

"Vertical"

6. $x^2 - 2x - 4y + 9 = 0$

$(x^2 - 2x + 1) = 4y - 9 + 1$

$(x-1)^2 = 4y - 8$

$(x-1)^2 = 4(y-2)$

$p = 1$

Vertex: $(1,2)$ Focus: $(1,3)$

Directrix: $y = 1$

plug $y=3$

$(x-1)^2 = 4(3-2)$

$\sqrt{(x-1)^2} = \sqrt{4}$

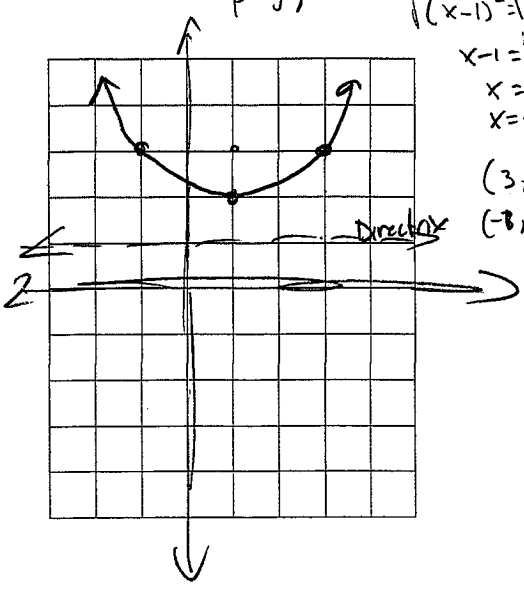
$x-1 = \pm 2$

$x = 2+1 = 3$

$x = -2+1 = -1$

$(3,3)$

$(-1,3)$



"Horiz"

7. $y^2 - 2y - 8x + 1 = 0$

$y^2 - 2y + 1 = 8x - 1 \quad +1$

$(y-1)^2 = \underset{4p}{8}x \quad p = 2$

Vertex: $(0,1)$

$p = 2$

Focus: $(2,1)$

Directrix: $x = -2$

plug in $x=2$

$(y-1)^2 = 8(2)$

$\sqrt{(y-1)^2} = \sqrt{16}$

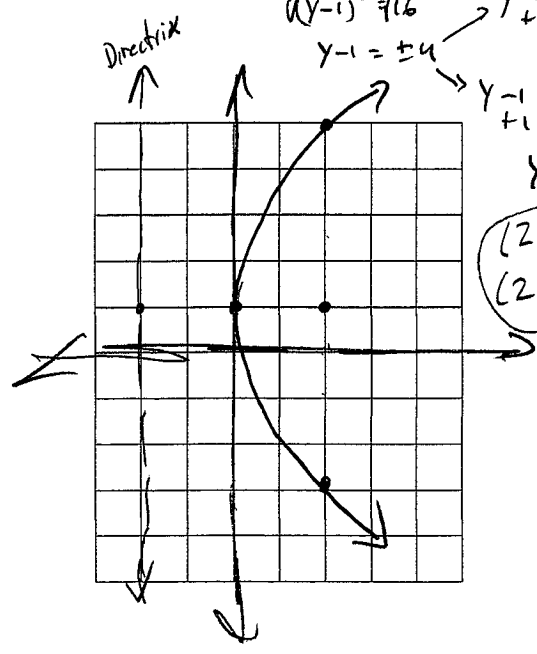
$y-1 = \pm 4$

$y-1 = 4$, $+1 \quad +1$, $y = 5$

$y-1 = -4$, $+1 \quad +1$, $y = -3$

$(2,5)$

$(2,-3)$



Find the solution to the system by graphing.

8. $\begin{cases} x = y^2 - 5 \rightarrow y^2 = x+5 \rightarrow \text{"Horiz" parabola} \\ x^2 + y^2 = 25 \rightarrow \text{circle} \end{cases}$

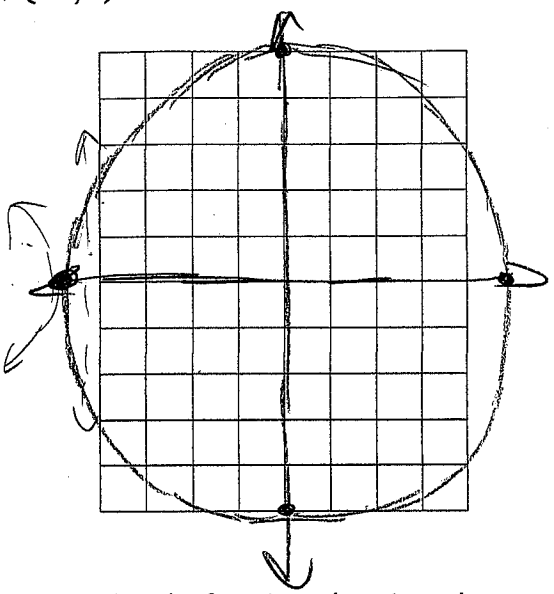
$y^2 = x + 5$

$y^2 = 1(x+5)$

$p = \frac{1}{4}$

Vertex $(-5, 0)$

Solution
$(-5, 0)$



9. $\begin{cases} (y-3)^2 = x-2 \rightarrow \text{Horiz. parabola} \\ x + y = 5 \rightarrow y = -x+5 \leftarrow \text{linear} \end{cases}$

$(y-3)^2 = x-2$

$(y-3)^2 = 1(x-2) \quad p = \frac{1}{4}$

vertex: $(2, 3)$

Focus: $(2.25, 3)$

Directrix: $x = 1.75$

$(y-3)^2 = 2.25 - 2$

$\sqrt{(y-3)^2} = \sqrt{0.25}$

$y - 3 = \pm 0.5$

$y = 3.5 \rightarrow (2.25, 3.5)$

$y = 2.5 \rightarrow (2.25, 2.5)$

$(3, 2)$?

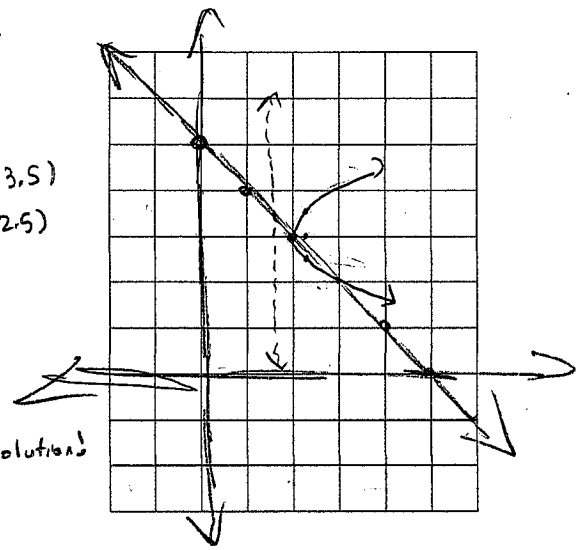
$(2-3)^2 = 3-2$

$(-1)^2 = 1$

$1 = 1$ ✓ Solution!

$3 + 2 = 5$ ✓

Solution
$(2, 3)$ ✓
$(3, 2)$ ✓



10. Determine the functions domain and range. $x = -4(y-1)^2 + 3$

$$\frac{x-3}{-4} = \frac{-4(y-1)^2}{-4}$$

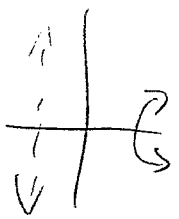
$y^2 = 4px$ means "Horiz" parabola

$(y-1)^2 = \boxed{-\frac{1}{4}}(x-3)$ → vertex $(3, 1)$

$$\frac{4p}{4} = \frac{-\frac{1}{4}}{4}$$

$p = -\frac{1}{16}$

quick sketch



From vertex

Domain: $[x, \infty)$

Range: $(-\infty, \infty)$

Domain
$[3, \infty)$

Range
$(-\infty, \infty)$