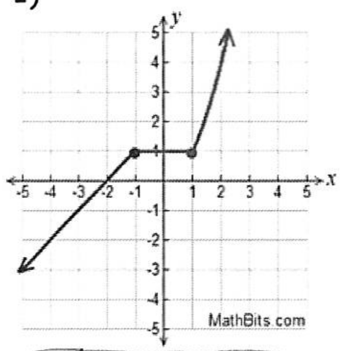
Warm-Up

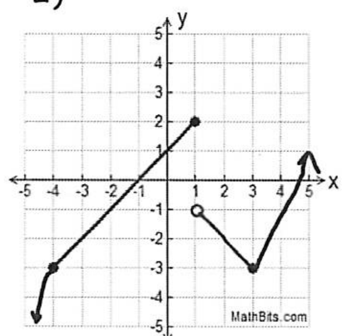
For each of the following, determine where the function is DISCONTINUOUS (if any).

1)



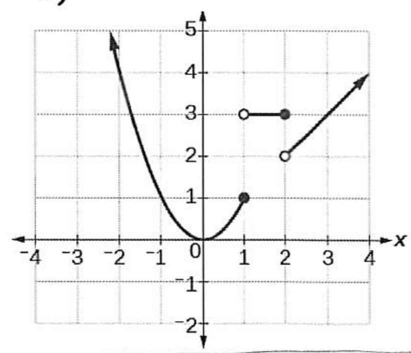
None, the function is continuous at all points.

2)



The function is discontinuous at 1

3)



The function is discontinuous at 1 & 2.

A function **f is continuous at a** when three conditions are satisfied.

1) f is defined at a; that is, a is in the domain of f, so that f(a) is a real number.

2) $\lim_{x \to a} f(x)$

3) $\lim_{x \to a} f(x) = f(a)$

Here is what the above means...

1) plug in the value, see if it results in a real number.

2) use the properties of limits to determine if the limit exist.

3) Does step 1 match step 2?

★ Sometimes look at the left & right sides to determine the limit.

Ex1) Determine whether the function f(x) is continuous

$$f(x)=\frac{2x+1}{2x^2-x-1}$$

A) at 2

1. $f(2)=\frac{2(2)+1}{2(2)^2-2-1}=\frac{4+1}{8-2-1}=\frac{5}{5}=1$

$f(2)=1$

2. $\lim_{x\to 2} f(x)=\lim_{x\to 2}\frac{2x+1}{2x^2-x-1}=\frac{\lim_{x\to 2} 2x+1}{\lim_{x\to 2} 2x^2-x-1}$

$=\frac{\lim_{x\to 2} 2(2)+1}{\lim_{x\to 2} 2(2)^2-2-1}=\frac{5}{5}=1$

3. $\lim_{x\to 2} f(x)=f(2)$ $1=1$ Match!

The function is continuous at 2

B) at 1

1. $f(1)=\frac{2(1)+1}{2(1)^2-1-1}=\frac{2+1}{2-1-1}=\frac{3}{0}=$ UND

Since $f(1)=$ UND, The function is discontinuous at 1.

* Sometimes you need to check if you can factor and simplify.

Ex2) Determine for what values of x, if any, the following functions is discontinuous.

$$f(x)=\begin{cases} x+2 & \text{if} \quad x\le 0 \\ 2 & \text{if} \quad 0<x\le 1 \\ x^2+2 & \text{if} \quad x>1 \end{cases}$$

Where should we look?

We should look where the function changes.

Check 0

1. $f(0)=2$

2. $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-} x+2=2$

$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+} 2=2$

$\lim_{x\to 0} f(x)=2$

3. $\lim_{x\to 0} f(x)=f(0)$

$2=2$ ✓ match!

Continuous at 0

Check 1

1. $f(1)=2$

2. $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-} 2=2$

Not a match $\lim_{x\to 1}=$ DNE

$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+} x^2+2=\lim_{x\to 1^+} 1^2+2=3$

Since the limit does not exist at 1, the function is discontinuous at 1